LINEAR SPRINGING MODEL – COMPARISONS OF DIFFERENT NUMERICAL MODELS

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ABSTRACT

This study presents comparisons among different numerical approaches for ship hydroelasticity problem, particularly focusing on linear springing phenomenon. Both Rankine-panel based time domain approach and wave-Green-function based frequency domain approach are cross compared with each other. For the time-domain approach, a higher-order Rankine panel method is used for the fluid motion around a flexible seagoing vessel. The motion of flexible hull is represented by Vlasov-beam based finite element method, and the motion equation is solved by Newmark-\(\beta\) method. For the frequency-domain approach, wave Green function method is employed in taking hydrodynamic forces acting on the body into consideration, and the motion of flexible body is solved by using finite element method as well. Solution of the motion equation is sought by modal decomposition method, where the unknown deformation field is expanded by superimposing its mode shapes, including six rigid body ones. Comparison between the two solutions is made for both a flexible stationary barge model as well as a modern merchant ship model. Based on the present comparative study, the pros and cons of each method are discussed.

KEYWORDS

Hydroelasticity; springing; Rankine-panel; wave Green function, Vlasov

1. INTRODUCTION

Since its very first tragic experiences from the bulk carriers in Great Lakes during late 60s, many research efforts have been devoted to the ship springing problem in order to obtain practical numerical tools to estimate this phenomenon, not to speak of the understanding of the physics behind it. Early pioneering work by Bishop and Price\textsuperscript{1} is the most widely recognized one amongst others related to this topic. The approach they took starts from the modal decomposition of the structural response, and all the other relevant hydrodynamic variables are handled within this framework by projecting the quantities to the modal space. Price and Temarel\textsuperscript{2} and Pedersen\textsuperscript{3} carried out extensive studies on the coupled horizontal bending and torsion response of container carrier taking into account warping stiffness. Jensen and Pedersen\textsuperscript{4} proposed the second order strip theory based on the relative motion strip theory concept. Later, Vidic-Perunovic\textsuperscript{5} extended the second order strip theory by being able to take into account the effect of cross-coupling terms of bichromatic incident waves, as well as the sum-frequency effect of monochromatic ones.

A recent contribution to ship hydroelasticity problem can be found in the works of Malenica...
et al. and Senjanovic et al\textsuperscript{6-9}. They used a frequency domain wave Green function (WGF) method combined with a finite element beam model which is able to capture both bending-torsion coupling and warping deformation. Verification through model basin tests was carried out with a segmented model composed of 12 separated pontoon-like hulls. A time domain approach which directly couples BEM and FEM was studied by Kim et al\textsuperscript{10}. They used higher order B-spline Rankine panel method (RPM) for the fluid flow and a Vlasov beam element for the structural response of the ship with thin-walled open section. The solution of motion equation was sought by direct time integration method, such as Newmark-type second order implicit scheme. Remy et al. carried out experiments on the flexible response of floating barge under both head and oblique waves\textsuperscript{11}.

In this study, both time and frequency domain solutions of linear ship springing problem are tackled and the solutions are cross compared. There are several important differences between the two models. In the time domain approach the so called direct FEM-BEM coupling is used while the frequency domain approach uses the modal decomposition method. At the same time the time domain method uses the Rankine panel method for solving the hydrodynamic boundary value problems, while the frequency domain method employs the wave Green function method. The last difference between the two methods concerns the structural model which is based on the Vlasov theory in the time domain method, while the advanced beam theory presented by Senjanovic et al. is used in the frequency domain method\textsuperscript{9}.

2. THEORETICAL BACKGROUND

2.1 Time domain approach

For the solution of fluid part, velocity potential, \( \phi(x, t) \), satisfying Laplace’s equation can be defined inside fluid domain by Eq.(1) under the assumption that fluid is inviscid, incompressible, and flow is irrotational,

\[
\nabla^2 \phi(x, t) = 0.
\]

\hspace{1cm} (1)

It is assumed that the total potential \( \phi(x, t) \) can be decomposed into three components: the steady basis potential \( \Phi(x, t) \), the incident potential, \( \phi_I(x, t) \) and the disturbed potential, \( \phi_d(x, t) \). The incident potential is given by linear gravity wave theory, and the basis potential is obtained using the double body formulation. For the solution of the unsteady disturbed potential, which is induced by dynamic motion of the floating body, proper boundary condition has to be introduced both on the free surface and the body surface, together with corresponding far field condition. Eq.(2) shows both kinematic and dynamic free surface boundary conditions, both of which have to be met on mean free surface.

\[
\frac{\partial \zeta_d}{\partial t} - (V - \nabla \Phi) \cdot \nabla \zeta_d = \frac{\partial^2 \Phi}{\partial z^2} \zeta_d \frac{\partial \phi_d}{\partial z} + (V - \nabla \Phi) \cdot \nabla \zeta_d
\]

\[
\frac{\partial \phi_d}{\partial t} - (V - \nabla \Phi) \cdot \nabla \phi_d = -\frac{\partial \Phi}{\partial t} - g \zeta_d + \left[ V \cdot \nabla \Phi - \frac{1}{2} \nabla \Phi \cdot \nabla \Phi \right] + (V - \nabla \Phi) \cdot \nabla \phi_d
\]

\hspace{1cm} (2)

where \( \zeta_d \) is disturbed wave elevation and \( V \) is ship’s forward speed.

The motion of the body in the fluid domain requires the body boundary condition to be met on the exact body location given by Eq.(3). Unlike the rigid body case, the body boundary condition has to be treated separately for each panel due to the arbitrariness of the deformation pattern of the flexible hull.
\[
\frac{\partial \phi}{\partial t} \Bigg|_{n} = \frac{\partial u}{\partial t} \cdot n - \frac{\partial \phi}{\partial n} + (V - \nabla \Phi) \cdot n \quad \text{on } S_n
\]  

(3)

where \( u \) is the deformation of the hydrodynamic panel induced by the structural deformation and \( n \) is the normal vector of the panel. The body boundary condition given in Eq. (3) needs to be further extended to the linearized one that should be met on its mean body condition. Timman and Newman\(^{13}\) used Taylor expansion to obtain the linearized body boundary condition to be met on its mean position as indicated by Eq. (4). Also, the steady-unsteady coupling term, which is often called ‘m-term’ is taken into account by relying on the modified Nakos approach\(^{14}\), namely

\[
\frac{\partial \phi}{\partial t} \Bigg|_{n} = \frac{\partial u}{\partial t} \cdot n - \frac{\partial \phi}{\partial n} + \left[ (u \cdot V) V - (u \cdot V) \nabla \Phi + ((V - \nabla \Phi) \cdot V) u \right] \cdot n \quad \text{on } S_n.
\]  

(4)

For the uniqueness of the solution, the radiation boundary condition at far field should be satisfied. In the present study, the radiation condition is realized by adopting the concept of artificial damping zone along the outer perimeter of the free surface panel\(^{15}\).

The solution of the above mentioned boundary value problem was sought by integral equation which starts from Green’s second identity. In doing so, the solution variables such as potential and its normal derivatives etc., are discretized in space by using bi-quadratic B-spline function so that continuity up to its second derivatives is guaranteed across each panel. Details can be found in the work by Nakos\(^{15}\).

For the solution of the structural part, Vlasov beam theory with 7 DOFs per node is followed which is able to take into account the effect of warping distortion\(^{16}\). The effect of warping becomes very important when the cross section of the ship is a thin-walled-open one, which is the case of modern large sized container carrier. Also, the effect of the coupling between horizontal bending and torsion which is caused by the gap between neutral axis and shear centre location, is taken into account. In addition, beam offset effect which is caused by the non-matching neutral, shear and mass centre along ship length direction is considered. Some important sectional constants that are introduced as a result of the cross section integration during energy formulation are second moment of inertias including polar one, shear centre, St.Venant torsional constant, warping constant, mass moment of inertias, and warping inertia and so on. Compact form of the structural equation is shown in Eq. (5), where M, C and K matrices denote mass, structural damping and stiffness matrices respectively. Structural damping matrix is constructed based on the Rayleigh damping approximation, i.e.

\[
MU + CU + KU = F
\]

\[
C = \alpha M + \beta K
\]  

(5)

Time integration of Eq. (5) is carried out by Newmark-\(\beta\) method, which is one of the most popularly used schemes in structural mechanics field. Artificial damping may be introduced during time integration by adjusting Newmark parameters to suppress very high frequency oscillation, which is not the physical behaviour of interest.

To obtain the solution from the above mentioned two coupled equations, an iterative method is used where solutions of two field equations are exchanged between them until the converged solution is obtained. Fixed point iteration with relaxation method is used to accelerate the convergence rate and the modified Aitken’s \(\delta^2\) process is used for the determination of optimum relaxation parameter. Eq. (6) shows how the iteration is performed within each time step, namely
\[
U_{i+1|z}^{k} = \mathbf{S}(p_{i+1|z}) \\
p_{i+1|z}^{k+1} = F(U_{i+1|z}^{k+1}, U_{i+1|z}^{k+1}, \phi_{j, i+1|z})
\]
iterate until \[|U_{i+1|z}^{k} - U_{i+1|z}^{k+1}| < \varepsilon\] (6)

where S and F represent the structural and fluid equations respectively. \(p\) is the fluid pressure, \(U\) is the structural deformation and \(\phi\) is the potential on the free surface. \(\varepsilon\) means convergence tolerance. Details can be found in the work by Kim\(^{14}\).

### 2.2 Frequency domain approach

The frequency domain hydroelastic model basically extends the motion representation of the well known rigid body seakeeping model with some additional modes of motion/deformation. We write:

\[
H(x, y, z, t) = \sum_{i=1}^{N} \xi_i(t) h'(x, y, z) = \sum_{i=1}^{N} \xi_i(t) \left[ h_i(x, y, z) i + h_i(x, y, z) j + h_i(x, y, z) k \right]
\]

where \( h'(x, y, z) \) denotes a general motion/deformation mode, which can be either rigid or elastic. The elastic modes are chosen as a series of the dry structural natural modes of the hull girder. An advanced thin wall girder beam finite element method is used for the structural analysis, as explained by Senjanovic et al\(^{8,9}\).

The total velocity potential \( \varphi \) is decomposed into the incident, diffracted and radiated components for every mode in the above decomposition:

\[
\varphi = \varphi_i + \varphi_D - i \omega \sum_{j=1}^{N} \xi_j \varphi^j
\]

where:
- \( \varphi_i \) incident potential,
- \( \varphi_D \) diffraction potential,
- \( \varphi^j \) \( j \)-th radiation potential,
- \( \omega \) encounter frequency.

The encounter frequency approach is used to include the effect of forward speed. This is a common approach as it is still very difficult to solve the boundary value problems (BVP) with the forward speed included correctly. Using the encounter frequency approach, the BVP reduces to:

\[
\begin{align*}
\Delta \varphi &= 0 & \text{in the fluid} \\
- \nu_e \varphi + \left( \frac{\partial \varphi}{\partial z} \right) &= 0 & z = 0 \\
\left( \frac{\partial \varphi}{\partial n} \right) &= V_e & \text{on } S_h \\
\lim_{R \to \infty} \left[ \nu_e R \left( \frac{\partial \varphi}{\partial R} - i \nu_e \varphi \right) \right] &= 0 & R \to \infty
\end{align*}
\]

where:
- \( S_h \) is the mean hull surface,
- \( R \) is the horizontal distance from the body,
- \( \nu_e \) is the encounter frequency based wave number \( \nu_e = \omega / g \).
\( n \) is the normal vector of the hull surface.

\( V_n \) denotes the normal velocity which is obtained after proper linearization of the original nonlinear body boundary condition:

\[
\frac{\partial \phi_n}{\partial n} = -\frac{\partial \phi}{\partial n} \tag{10}
\]

\[
\frac{\partial \phi_k}{\partial n} = h' n + \frac{1}{\omega_k} \left[ (W \cdot \nabla) h' - (h' \cdot \nabla) W \right] \tag{11}
\]

with \( W = U \nabla (\phi - x) \), \( \phi \) being the double body flow and \( U \) being the forward speed. In the present calculations, the uniform flow approximation is used, so the steady velocity reduces to \( W = -Ui \).

The above boundary value problems are solved numerically using constant source distribution over the hydrodynamic mesh panels. The pressure on the hull can then be calculated by Bernoulli’s equation:

\[
p = i\omega \rho \varphi - \rho W \nabla \varphi. \tag{12}
\]

The diffraction and radiation coefficients are obtained by projecting the pressure on each mode shape and integrating the over the wetted surface, which results in:

\[
F_{\text{diff}} = i\omega \rho \int \left( \varphi_i + \varphi_n + \frac{i}{\omega} W \nabla (\varphi_i + \varphi_n) \right) h' n dS; \tag{13}
\]

\[
A = \rho \Re \left( \int \left( \varphi_i + \varphi_n \right) h' n dS \right); \tag{14}
\]

\[
B = \rho \omega \Re \left( \int \left( \varphi_i + \varphi_n \right) h' n dS \right). \tag{15}
\]

The following coupled frequency domain motion equation can then be written:

\[
\left( -\omega^2 \left( [m] + [A] \right) - i\omega \left[ B \right] + \left[ C \right] \right) \{\xi\} = \{F_{\text{diff}}\} \tag{16}
\]

where:

\([m]\) is the modal genuine mass matrix,

\([k]\) is the modal structural stiffness matrix,

\([A]\) is the hydrodynamic added mass matrix,

\([B]\) is the hydrodynamic damping matrix,

\([C]\) is the hydrostatic stiffness matrix,

\(\{\xi\}\) is the modal amplitudes vector,

\(\{F_{\text{diff}}\}\) is the modal excitation vector.

The solution of the above equation gives the motion amplitudes and phase angles \(\xi_i\) and the problem is formally solved. Note that the motion equation includes both 6 rigid body modes and a certain number of elastic modes.
3. COMPUTATIONAL RESULTS

Three different analysis models were chosen for validation purpose. First one is a stationary barge model made of 12 separate pontoons tied together using a flexible backbone. The other two are real modern merchant ship models, namely a LNG carrier and a container carrier.

3.1 Flexible barge model
Numerical computation was carried out for a flexible ship-like barge which was experimented by Remy et al. Fig. 1 shows the segmented model used in their experiments together with the details of the model. The barge is made of 12 segmented pontoons, and each pontoon is 0.19m long, 0.6m wide and 0.25m deep. Overall barge length is 2.445m and its draft is 0.12m. The centre of gravity of each pontoon is located 163mm above the keel line, and the radius of gyration for roll is 225mm. The foremost pontoon is slightly modified as shown in Fig.1(b). 12 segmented pontoons were tied together with steel rod placed on deck whose cross section is 10mm × 10mm square. Details of the experiment can be found in the work by Remy et al.

Fig. 2 shows vertical motion RAOs at each measuring point. The comparison between time and frequency domain solutions ended up with a fairly good correspondence with experimental data. RAOs from time domain approach were obtained by transforming the time history of the corresponding quantity using standard Fourier analysis.

Figure 2: Vertical motion RAOs at each measuring point under $\beta=180^\circ$
(Dot-Experiment, Black solid-Frequency domain, Red broken-Time domain)
3.2 LNG carrier

The length between perpendiculars of the ship is 303m and its draft at full load condition is 11.9m. The ship is assumed to travel with even trim and speed is 10.03m/sec, which corresponds to Fn of 0.18. Fig.3 shows hull form of the ship and the distribution of 2nd moment of inertia, together with mass per length, along ship’s length. Since vertical bending is the lowest natural mode of this ship, only 180° heading angle is considered in this case.

The dry eigenvalue analysis results showed that both programs gave almost identical natural frequency for two node vertical bending mode, i.e. the time domain code giving 7.48 rad/sec, and the frequency domain code 7.47 rad/sec. Fig.4 shows the sectional load RAO comparisons, for both vertical bending moment and vertical shear force, between the two codes. (a) and (c) are the results of the rigid body and (b) and (d) are that of the flexible body. When the body is assumed to be rigid, the response peaked once around heave resonance frequency then calmed down afterwards. However, when the body is flexible, additional sharp peak can be seen in the high frequency range due to the resonance vibration (springing). Wet natural frequency of two node vertical bending mode, where the sharp peak appears, turned out to be 5.09 rad/sec. This corresponds to the encounter frequency of the incoming wave of 1.8 rad/sec.

3.3 Container carrier

The length between the perpendiculars of the ship is 348m and the draft is 15.61m under full load condition. The ship speed is 12.7 m/sec and the considered heading angles are 180°, 120°. In case of the container carrier, the response under oblique wave needs to be focused on because the mode shape with the lowest natural frequency of the ship is the torsion related one. Fig.5 shows hull form and some examples of cross section data, among about 15 in total, including warping constant/inertia, shear centre and St.Venant torsional constant and so on. Most of the cross section is open except for some short span near the accommodation area as well as fore and aft peaks where the cross section is closed. At these closed section areas, torsional rigidity increases a lot and warping is highly restrained.
Table 1 summarizes the computational results on dry natural frequency of the ship. The lowest mode shape turned out to be a simple torsional mode with negligible horizontal bending followed by a coupled horizontal bending and torsion mode (HB+T). The third and sixth modes are 2 and 3 node vertical bending modes. Overall correspondence between the two codes is fairly good even though higher HB+T modes show some discrepancy.
Fig. 6 shows both vertical bending moment (VBM) and vertical shear force (VSF) RAO under 180° heading angle. For both cases, the correspondence between the two different approaches is very good, except for the magnitude at the resonance frequency. The reason why time domain approach results in a slightly lower value of magnitude at resonance frequency may be ascribed to the different way of handling the structural damping. Even though the same structural damping ratio was used for both cases, the time domain approach predicts a smaller peak at the resonance frequency since the direct integration in time domain uses a Rayleigh damping method which is not exactly same as the modal damping method which is used in frequency domain approach. The first wet resonance of two node vertical bending takes place at a wave frequency value of 1.1 rad/sec, which corresponds to an encounter frequency of 2.7 rad/sec. The second peak is located around 1.75 rad/sec, whose encounter frequency equals to 5.73 rad/sec. Unlike the first resonance case, this second peak shows a lower level of VBM along with a higher VSF. This can be understood easily by the mode shape of three node natural vibration mode, where bending moment peaks at both quarter points whereas shear force peaks in the middle.

![Figure 6: Sectional load RAO at amidships (β=180°)](image)

Fig. 7 shows a sectional load RAO comparison between the two approaches together with rigid body response when wave heading is 120°. Line with dots denotes the result of the rigid body, where dynamic effect due to ship flexibility as well as hydroelasticity effect is not taken into account. As expected, both rigid and elastic results are close to each other in the low frequency range, but towards the high frequency range the gap starts to increase due to flexibility of the hull. Again, VBM RAOS between time and frequency domain approaches compare well in this oblique wave case. However, other sectional loads show some discrepancy. Focusing on the torsional moment (TM) RAO, two results predicted similar resonance frequency but the magnitude difference at these resonance frequencies is rather important. Since horizontal bending
(HB) as well as warping induced bimoment (BM) is coupled with torsional response, those RAOs also show some discrepancy between the two approaches.

To investigate the difference between the two methods in more detail, further comparison was made for the torsional moment RAO. In Fig.8, the torsional moment RAO is decomposed into quasi-static and dynamic parts. Quasi-static response is what can be obtained by conventional rigid body ship motion analysis and dynamic response is the remainder, which is to be covered by flexibility of the hull. In other words, the quasi-static response is wholly governed by wave induced load whereas dynamic response is governed by the dynamic vibration with hydroelasticity effect. Fig.8(a) shows that the quasi-static response of the two approaches compares well, but the discrepancy can be observed in the dynamic response as shown in Fig.8(b). Main source of the difference in dynamic response may be ascribed to the different structural behaviour of the two approaches. As is summarized in Table 1, the correspondence of the dry natural frequencies on the torsion related modes is not as good as that on vertical bending mode, and this may lead to the rather unsatisfactory sectional load comparison such as TM, HBM and BM, all of which are coupled together. One of the most difficult parts when handling this complicated structural behaviour in the framework of beam theory is that the solution tends to vary depending on the way the effect of the strong transverse structural members is treated and also the way kinematic compatibility of warping deformation between open and closed section is imposed.

Figure 7: Sectional load RAO at amidships (β=120°)
4. CONCLUSIONS

Based on the comparative study between time domain RPM and frequency domain WGF approach about linear springing problem, the following preliminary conclusions can be drawn:

- For stationary simple barge model, both approaches produce almost identical results. Numerical results also showed good correspondence with experimental data.
- The LNG carrier which was modelled as a closed section beam in both methods, was analyzed in head wave condition. It turned out that both results compared well in terms of VBM and VSF.
- The container carrier which was modelled as a thin-walled open-closed section beam, was considered in both head and oblique waves. Dry eigenvalue analysis results for vertical bending mode are almost identical between the two programs; however, small deviation was found when it comes to the modes which include torsional response.
- Symmetric response such as VBM and VSF showed good correlation between the two approaches, while antisymmetric response such as TM and HBM did not. When decomposed into quasi-static and dynamic parts, the main source of the difference between the two cases is the dynamic part which is mainly linked to the different dynamic characteristics of the two structural models.

In order to clarify the source of the difference between the two methods, further investigation on the details of the finite element formulation of thin-walled open section girders is required. In addition, possible differences in hydrodynamic models between the two methods, such as the formulation of basis potential, steady-unsteady coupling term and the effect of mesh resolution deserves to be investigated more carefully.

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