FATIGUE ANALYSIS OF AN FPSO UNDER OPERATIONAL SEA STATES WITH MULTIMODAL SPECTRA

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ABSTRACT

This paper presents a validation of theoretical formulas for the assessment of fatigue damage induced by a multimodal wave spectrum. Those formulas, denoted Iterative Component Addition (ICA), were set up to provide a conservative estimate and use the individual damages of the components of the spectrum. Their objective and main interest is a drastic reduction of computation time, especially for complex (multimodal) wave loading conditions. They are validated on an actual application, which is the assessment of the fatigue damage induced by the wave bending moment of an FPSO hull girder subjected to wave loading in a West Africa area. That damage is computed according to two different procedures, one with the conventional method, taken as a reference, and the other using the ICAs formulas. The results are compared and discussed. They show that the use of ICA formulas provides a reasonably conservative estimate and allows significant savings of computational time.

NOMENCLATURE

- $\omega$: frequency in radian per second
- $H_S$: significant wave height
- $T_p$: peak period
- $\theta$: direction
- $W$: wave loading spectrum
- $R$: structural response spectrum
- $\eta_s$: irregularity ratio of a signal, which is the ratio of zero-crossing number over turning point number
- $N$: number of cycles
- $S$: cycle range
- $\beta_l$: ratio $N_l/N_s$
- $\beta_h$: ratio $N_h/N_s$
- $D$: fatigue damage
- $\lambda_0$: spectral moment of order zero or variance
- $m$: slope of the S-N curve
- $\bar{\sigma}$: intercept of the design S-N curve with the log N axis
- $S_c$: fatigue limite at $10^7$ cycles
- $Q$: damage computations number (measure of the computation time)
- $\Gamma(\cdot)$: upper incomplete Gamma function
- $\gamma(\cdot)$: lower incomplete Gamma function
- RAO: Response Amplitude Operator
- LCF: Low Cycle Fatigue
- HCF: High Cycle Fatigue
- SCF: Stress Concentration Factor
- MS: main swell
- SS: secondary swell
- WS: wind sea

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1 INTRODUCTION

The conventional method for fatigue damage assessment of offshore mechanical structures can be very expensive and time consuming. One needs to compute the fatigue damage for any sea state that might occur during the lifetime of the structure. Each damage computation requires also many steps including structural response computation, rainflow counting, and elementary damage assessment. That procedure should be repeated for many simulations of each sea state time series. Moreover, the cost of the analysis increases when complex (multimodal) wave loading conditions have to be considered, as it is the case in West Africa. For such locations, new methods have been set up to describe the metocean climate as combinations of several (up to 3 or 4) wave systems that are categorized as swell or wind sea [1]. As a consequence, the spectrum exhibits several peak frequencies for most sea-states (85% in data of table 1 later below) and a very large number of sea states have to be considered. In this case, fatigue computations become significantly more complex than for sea states corresponding to a unimodal spectrum.

In a previous paper [2], the authors have set up formulas for the estimation, in a conservative manner, of the fatigue damage due to combinations of several wave systems in terms of damages corresponding to each system taken independently. Those formulas allow a significant computational time reduction. In fact, the fatigue damage assessment with all the required steps is carried out only for the individual wave systems, the number of which is much lower than that of all their possible combinations. Then, the damage of any combination is evaluated by a straight application of the formulas. Those formulas called Iterative Components Addition (ICA) were shown to meet a large straight application of the formulas. Those formulas called Iterations. Then, the damage of any combination is evaluated by a system, i.e. and reasonable conservatism. Moreover, those formulas can accept a two slope S-N curve and can be iteratively applied when more than two components need to be combined, as it is often the case in practice.

The issue now is to validate those formulas on a realistic fatigue design application. The present study focuses on the particular case of a structural response represented by a linear dynamic system, i.e. which can be obtained with Response Operator Amplitudes (RAOs). To validate the practical use of those formulas, the authors compute and analyse the fatigue damage over the lifetime of a structure according to two different approaches:

- **Reference case**: It is assumed that any sea state in the database is repeated on a periodic basis such that the whole set covers the design lifetime. Hence, considering a set of three-hour sea states, a sufficient number of time series of the structural response is simulated for each recorded sea state so that the total scales to the entire design lifetime, then the corresponding damage is computed and accumulated.

- **ICAs formulas**: A statistical description of the sea state data is constructed, grouping spectral parameters of the identified wave systems into discretized classes. Then their respective damages are computed by simulation for the mean values of parameters in each class. Finally, the damages of all combinations that can occur are estimated by ICAs formulas and added, weighted by their respective occurrence probabilities, and the resulting damage is scaled to the design fatigue lifetime.

The two approaches are applied on an industry actual case, namely the fatigue damage due to the wave induced bending moment of an FPSO hull girder subjected to wave loading in a West Africa area. Their respective results are analysed, compared and discussed.

2 ITERATIVE COMPONENT ADDITION FORMULAS

Let us consider a random signal with a bi-modal power spectral density (Figure 1), which is the sum of a low frequency spectrum and a high frequency spectrum. It may represent the stress field or an equivalent stress at a critical point in the mechanical structure under study.

![Figure 1. Bi-modal spectrum.](image)

Two Iterative Component Addition formulas, denoted $ICA_1$ & $ICA_2$, have been proposed by Olagnon & Guédé (2008) [2], and they aim at estimating the Rainflow fatigue damage of the signal from the damages $D_h$ and $D_l$ of its high and low frequencies components, calculated separately. They are based on a partition of the set of local extrema of the global signal into two separate subsets $A$ and $B$ (see figure 2). Subset $B$ contains the maximum and minimum local extrema between two successive zero-crossings of the low-frequency signal, while subset $A$ contains all the remaining turning points.

From the mathematical formulation of the rainflow counting [3], both subsets are shown to be stable by rainflow counting.
Thus, the total damage is the simple sum of the damages of the subsets \( A \) and \( B \) taken separately: \( D = D_A + D_B \).

Now, let us consider a two slope S-N curve with parameters \( m_1 \) and \( \pi \) for low-cycle fatigue regime (LCF) and \( m_2 \) and \( \pi \) for the high-cycle fatigue regime (HCF) with a threshold \( S_c \).

### 2.1 Damage of subset \( A \)

The damage of subset \( A \) reads:

\[
D_A = \frac{N_A}{\pi_1} \int_{S_c}^{\infty} S_A^{m_1} p(S_A) dS_A + \frac{N_A}{\pi_2} \int_{0}^{S_c} S_A^{m_2} p(S_A) dS_A \quad (1)
\]

Compared to high-frequency rainflow ranges, those of \( A \) are lower in terms of amplitude and number (addition of the low frequency component reduces the cycle ranges). The number of cycles \( N_A \) can be approximated by:

\[
N_A = \frac{1 - \beta_l}{\beta_h} N_h \quad \text{where} \quad \beta_l = \frac{N_l}{N_a}; \quad \beta_h = \frac{N_h}{N_a} \quad (2)
\]

Assuming that ranges \( S_A \) have the same distribution as high frequency component ranges \( S_h \) (i.e. \( p(S_A) dS_A = p(S_h) dS_h \)) and using the fact that \( S_A \) is lower than \( S_h \), a conservative estimate of \( D_A \) is achieved by:

\[
D_A = \frac{1 - \beta_l}{\beta_h} D_h \quad (3)
\]

### 2.2 Damage of subset \( B \)

The damage of subset \( B \) reads:

\[
D_B = \frac{N_l}{\pi_1} \int_{S_c}^{\infty} S_B^{m_1} p(S_B) dS_B + \frac{N_l}{\pi_2} \int_{0}^{S_c} S_B^{m_2} p(S_B) dS_B \quad (4)
\]

Under the narrow-band assumption for low-frequency component, the damage associated to the low-frequency component reads:

\[
D_l = D_l^{LCF} + D_l^{HCF}
\]

where

\[
D_l^{LCF} = \frac{N_l}{\pi_1} (2 \sqrt{2 \lambda_{0.1}})^{m_1} \left( 1 + \frac{m_1}{2} \frac{S_c^2}{8 \lambda_{0.1}} \right) \quad (5)
\]

\[
D_l^{HCF} = \frac{N_l}{\pi_2} (2 \sqrt{2 \lambda_{0.1}})^{m_2} \gamma \left( 1 + \frac{m_2}{2} \frac{S_c^2}{8 \lambda_{0.1}} \right) \quad (6)
\]

\( \Gamma \) and \( \gamma \) are respectively the incomplete 'upper' and 'lower' gamma functions and \( \lambda_{0.1} \) is the variance of the low-frequency component of the signal.

When one replaces the respective expressions of the ratios \( N_l/\pi_1 \) and \( N_l/\pi_2 \), derived from Eq.(5) and Eq.(6) into Eq.(4), it comes out:

\[
D_B = \frac{Y^{LCF}}{G} \cdot D_l^{LCF} + \frac{Y^{HCF}}{g} \cdot D_l^{HCF} \quad (7)
\]

with

\[
Y^{LCF} = \int_{S_c}^{\infty} S_B^{m_1} p(S_B) dS_B
\]

\[
Y^{HCF} = \int_{0}^{S_c} S_B^{m_2} p(S_B) dS_B
\]

\[
G = (2 \sqrt{2 \lambda_{0.1}})^{m_1} \Gamma \left( 1 + \frac{m_1}{2} \frac{S_c^2}{8 \lambda_{0.1}} \right)
\]

\[
g = (2 \sqrt{2 \lambda_{0.1}})^{m_2} \gamma \left( 1 + \frac{m_2}{2} \frac{S_c^2}{8 \lambda_{0.1}} \right)
\]

One can see that the distribution \( p(S_B) \) needs to be known to compute the damage of subset \( B \).
2.3 Approximation of the distribution of ranges in 8

Two conservative approximations are suggested for the distribution of \( S_\beta \). Both approximations use the fact that the local maxima of a standard Gaussian process follow a Rice distribution. One approximation reads:

\[ Y_1(S) = \frac{\sqrt{\pi} n_p}{2 \beta_1} \]  

(8)

\[ Y_1(S) = \frac{dY_1}{dS} = n_p f \left( \frac{1}{\beta_1} \right) \]  

(9)

where, \( f_p(S) \), the distribution of the positive local maxima of the global signal is:

\[ f_p(S) = \frac{f(S) - f(0)}{f(\infty) - f(0)} = \frac{2f(S) + \eta_\beta}{\eta_\beta + 1} \]  

(10)

where \( \eta_\beta \) is the irregularity ratio of the total signal, and the average number of those maxima between two successive minima of the low-frequency component of the signal is:

\[ n_p = \frac{\eta_\beta + 1}{2 \beta_1} \]  

(11)

Approximation \( Y_2(S) \): It assumes that \( S_\beta \) are the largest possible local maxima of the global signal. Having noted that the mean number of local maxima between two successive minima of the low-frequency signal is fixed to its mean value. Thus the first approximation reads:

\[ Y_2(S) = 1 - \frac{1}{\beta_1} (1 - f(S)) \]  

for \( S \geq \delta_2 \)  

(12)

\[ Y_2(S) = \frac{dY_2}{dS} = \frac{f(S)}{\beta_1} \]  

for \( S \geq \delta_2 \)  

(13)

where \( \delta_2 = \max \left( 0, \frac{1}{\beta_1} (1 - \beta_1) \right) \)

The \( ICA_1 \) & \( ICA_2 \) formulas are respectively derived from the above approximations of \( p(S_\beta) \). It may be also noted that they are relatively simple formulas as they depend only on the parameters \( S_\beta, \eta_\beta, \beta_1 \) and \( \beta_2 \) and on the damages \( D_0 \) and \( D_1 \). Moreover, they can be iteratively applied with spectra having more than two peak frequencies. For example, in the case of a spectrum with three components, the combination of the two components with the highest peak frequencies is taken as the high frequency component of the global signal. They are first estimated with the ICA formulas. Then, the resulting high-frequency component damage is used in a second application of ICA with the lowest frequency component. This iterative application of ICA formula is made possible by the fact that no assumption is required on the shape of the high-frequency component, only the low-frequency component is supposed to be narrow-band.

The ICA method have been investigated [5]. The ICA method was found very robust, including case where the high and low frequency domains were adjacent, and even overlapping, i.e. beyond the limits of the separation in 8 and 3 that was used to build the method. Such situations are indeed routinely met in a multimodal description of sea-states, but are cases where other "dual band" formulas tend to diverge.

3 FATIGUE DAMAGE ASSESSMENT OF AN FPSO HULL STRUCTURE

3.1 Problem statement

The industrial application is based on an actual FPSO design at a West Africa location. The wave loads are computed from the set of metocean data provided by the operator. This set contains 8040 records of directional spectral parameters of sea state wave systems (e.g. significant height, \( H_\text{s} \), peak period, \( T_\text{p} \) and direction \( \theta \)). Assuming that a stationary sea state lasts 3 hours, this is corresponding to a duration of 2 years, 9 months and 15 days. Recorded sea states have each at most three wave systems categorized as either main swell (MS), secondary swell (SS) or wind sea (WS). That classification is based on criteria defined by the operator [6]. It allows to identify the various combinations of wave systems that may occur (see Table 1).

We focus in this study on the fatigue due to Vertical Wave Bending Moment (WWBM) at the midship of an FPSO. That structural response is assumed to be linear, and thus fully defined by the directional RAOs of stress range in a particular detail, taken proportional to WWBM. Hence, given a wave loading spectrum \( W(\omega, \theta) \) and the RAO, the response spectrum is given by:

\[ R(\omega, \theta) = \int_{\theta} \vert RAO(\omega, \theta) \vert^2 \cdot W(\omega, \theta) d\theta \]  

(14)

Note that in the present study, the directional spreading is ne-
Table 1. Proportions of the combinations.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Number</th>
<th>Frequency [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS only</td>
<td>1212</td>
<td>15.07</td>
</tr>
<tr>
<td>MS + SS</td>
<td>2740</td>
<td>34.08</td>
</tr>
<tr>
<td>MS + WS</td>
<td>1536</td>
<td>19.10</td>
</tr>
<tr>
<td>MS + SS+ WS</td>
<td>2549</td>
<td>31.70</td>
</tr>
<tr>
<td>no MS (i.e. WS only)</td>
<td>3</td>
<td>0.05</td>
</tr>
<tr>
<td>Total</td>
<td>8040</td>
<td>100</td>
</tr>
</tbody>
</table>

glected, and the wave spectrum is supposed to be unidirectional in its peak direction (i.e. \( W(\omega, \theta) = W(\omega) \cdot \delta_{\theta = 0} \)). This restriction does not affect the relevance of the analysis results for the issue considered here.

The fatigue strength criteria are based on Bureau Veritas rules [7]. A fatigue design life of 100 years is considered. The values used for the S-N curve parameters are shown in table 2.

<table>
<thead>
<tr>
<th>( \log_{10} \sigma_f )</th>
<th>( m_1 )</th>
<th>( \log_{10} \sigma_f )</th>
<th>( m_2 )</th>
<th>( S_c ) [MPa]</th>
<th>SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.261</td>
<td>3</td>
<td>14.101</td>
<td>5</td>
<td>26.32</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 2. SN curve parameters

### 3.2 Fatigue analysis

To validate the use of ICAs formulas, the fatigue damage of the FPSO is computed according to two different procedures:

- a reference calculation, which is a complete fatigue damage assessment based on the recorded metocean data.

- an ICAs-based method, which uses the ICAs formulas.

For the reference calculation, it is assumed that any sea state in the database is repeated on a periodic basis such that the whole set covers the design lifetime. Hence, assuming that a sea state lasts 3 hours and from the number of actual data (8040), 36 occurrences of each sea state are needed to cover the lifetime of 100 years. Then the damages of those 36 occurrences are estimated by time simulations. For that, we generate 36 3-hour time series instances of the sea state under consideration and compute the damage of each time series, instead of computing the damage of one generated time series and multiplying it by 36. We do so to keep the random effect of the spectrum of any sea state. Having longer records would be of course desirable to improve the accuracy of the prediction, in an actual design. This is however not affecting the comparisons of methods. On the other hand, this repetition is amply covering the variability of damage when computed from a time series. A total amount of \( Q_{\text{ref}} = 36 \cdot 8040 = 289440 \) time series are generated. For each of them, the structural response is computed, rainfall counting is performed and the corresponding damage \( D_k \) is estimated based on the Miner rule [8]. Those damages are finally cumulated to get the total damage \( D_t \):

\[
D_{t, \text{ref}} = \sum_{k=1}^{Q_{\text{ref}}} D_k
\]

Thus \( Q_{\text{ref}} = 289440 \) computations of damage by simulation are carried out, which is very expensive in terms of computation time.

To use the ICAs formulas, statistics of the metocean data are constructed. These statistics are based on a discretization of the wave systems parameters. In particular, \( H_3 \) is discretized into 7 classes, while \( T_p \) and \( \theta \) use respectively 23 and 16 classes. With that discretization, 312 triplets \((H_3, T_p, \theta)\), defining each observed wave system component, appear in the sea state data (i.e. any sea state is the combination of at most three elements of this set of 312 triplets). Analysing the 8040 sea states, it is found that \( Q_c = 3877 \) different combinations do actually occur, and their probabilities of occurrence are computed.

To apply ICAs formulas, one needs to compute first by simulation the representative damage of each element of the set of 312 wave system components. If 10 time series are generated to compute each representative damage, a total amount of \( Q_{\text{ICA}} = 312 \cdot 10 = 3120 \) time series are generated, which is almost one hundred times less than for the reference calculation. Once the representative damages of the wave systems individual components are computed, the damage of any of the \( Q_c \) combinations that can occur is computed by a straight application of the ICAs formulas. Then, the total damage is given by:

\[
D_{t, \text{ICA}} = Q_{\text{ref}} \cdot \sum_{k=1}^{Q_c} p_k \cdot D_{k, \text{ICA}}
\]

where \( D_{k, \text{ICA}} \) is the mean damage of the \( k \)-th combination and \( p_k \), its occurrence probability.

### 3.3 Results & discussion

The total fatigue damage obtained with the different methods is shown in tables 3 & 4. The damage obtained using the ICA formula on every (8040) observed sea states without any discretization of the parameters values are also presented in order to estimate separately the effects of discretization and of the
ICA formula. For the particular case analysed here (vessel Vertical Wave Bending Moment) the sea states containing only a main swell produce almost half of the total damage and the sea states with two swells contribute to the third of the total damage. Hence, the sea states containing only swells produce 80% of the total damage.

The damages obtained by the ICA-based method both with and without discretization are slightly higher than the reference values. The levels of conservatism as percentages of exceedance of the reference calculation are shown in table 5. The ICA-based procedure has a level of conservatism lower than 20%, which is reasonable. The level of conservatism of the application of ICA without discretization is much lower, and varies from 1% to 6%. This shows that the discretization is responsible for the largest part of the conservatism with respect to the reference calculation.

As indicated above, the ICA-based methods need one hundred times less simulations-based assessments of fatigue damages than the reference calculation, which leads to a significant saving of computational time. In addition, under the stationarity assumption of the 3 hours time series the scatter of any sea state damage is very low and one could reduce the number of simulations required for an accurate estimation of the representative values of the damages. In this way, the saving of time will be even higher.

4 Conclusion

In this study, the Iterative Components Addition formulas are validated on an actual industrial application. This application involves complex (multimodal) wave loading conditions such as those of West Africa metocean climate and a linear structural response (e.g. Vertical bending moment). A complete damage assessment is carried out using ICA formulas. The whole procedure is compared to a reference solution obtained by the conventional method in terms of accuracy and computational times. It is shown that ICA-based method provides a reasonably conservative estimate with a significant saving of time as expected. One should still note that at this stage the method requires that no non-linear interaction occur between the structural responses to the wave spectral components of each sea state taken individually.

REFERENCES

<table>
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<tr>
<th></th>
<th>$d_t$</th>
<th>$d_t \text{,ICA}_1$</th>
<th>$d_t \text{,ICA}_2$</th>
<th>$d_t^\ast \text{,ICA}_1$</th>
<th>$d_t^\ast \text{,ICA}_2$</th>
<th>Prop. damage</th>
<th>Prop. lifetime</th>
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<td>MS ONLY</td>
<td>43.4</td>
<td>49.1</td>
<td>49.1</td>
<td>43.4</td>
<td>43.4</td>
<td>46.73%</td>
<td>15.07%</td>
</tr>
<tr>
<td>MS + SS</td>
<td>33.6</td>
<td>35.7</td>
<td>35.8</td>
<td>33.9</td>
<td>34.0</td>
<td>36.11%</td>
<td>34.08%</td>
</tr>
<tr>
<td>MS + WS</td>
<td>7.4</td>
<td>8.1</td>
<td>8.1</td>
<td>7.4</td>
<td>7.4</td>
<td>7.93%</td>
<td>19.10%</td>
</tr>
<tr>
<td>MS + SS + WS</td>
<td>8.6</td>
<td>10.1</td>
<td>10.2</td>
<td>9.0</td>
<td>9.1</td>
<td>9.23%</td>
<td>31.70%</td>
</tr>
<tr>
<td>Total</td>
<td>92.9</td>
<td>103.1</td>
<td>103.3</td>
<td>93.7</td>
<td>93.8</td>
<td>100%</td>
<td>100%</td>
</tr>
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* without discretization

Table 3. Fatigue damage.

<table>
<thead>
<tr>
<th></th>
<th>lifetime</th>
<th>Ref. Calc.</th>
<th>$ICA_1$</th>
<th>$ICA_2$</th>
<th>$ICA_1^\ast$</th>
<th>$ICA_2^\ast$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lifetime</td>
<td>106.7</td>
<td>96.2</td>
<td>96.0</td>
<td>105.8</td>
<td>105.6</td>
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* without discretization

Table 4. Fatigue lifetime

<table>
<thead>
<tr>
<th></th>
<th>in %</th>
<th>$ICA_1$</th>
<th>$ICA_2$</th>
<th>$ICA_1^\ast$</th>
<th>$ICA_2^\ast$</th>
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<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>MS ONLY</td>
<td>13.2</td>
<td>13.2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>MS + SS</td>
<td>6.4</td>
<td>6.6</td>
<td>1.1</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>MS + WS</td>
<td>9.9</td>
<td>10.0</td>
<td>0.2</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>MS + SS + WS</td>
<td>18.1</td>
<td>19.2</td>
<td>4.7</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>all sea states</td>
<td>10.9</td>
<td>11.1</td>
<td>0.8</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

* without discretization

Table 5. Conservatism level.