Evaluation of Rule-Based Fatigue Design Loads Associated at a New Probability Level

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ABSTRACT

The paper presents a comprehensive analysis on the contribution of wave loads at different probability level of a Weibull distribution and typical S-N curves to fatigue damage and its sensitivity to the shape parameter. It shows that the probability level at 10^{-2} is most suited to evaluate the reference value for rule-based fatigue loads and discrepancy due to inaccuracy of the shape parameter is minimized. The application of new rule-based fatigue loads associated with 10^{-2} probability level will then provide the most reliable results in the fatigue analysis.

KEY WORDS

Rules, Design loads, Fatigue, Weibull, Shape factor

INTRODUCTION

To compute the fatigue damage on a structural detail is more complex than to assess the extreme stress, as it depends on the stress range histogram, which describes all the stress cycles encountered during the ship’s life. This histogram is then combined with the appropriate S-N curve to compute the fatigue damage according to the Palmgren-Miner rule.

\[ D = \sum n(\Delta \sigma) \quad (1) \]

where \( n(\Delta \sigma) \) is the number of stress cycles of range \( \Delta \sigma \), and \( N(\Delta \sigma) \) is given by the S-N curve, which may be defined by several slopes \( m_i \):

\[ N(\Delta \sigma) = \frac{K_i}{\Delta \sigma^{m_i}} \quad \text{for} \quad S_{0i} < \Delta \sigma < S_{b_i} \quad (2) \]

In the rule the long-term cumulative distribution of stress ranges is described by a 2 parameters Weibull distribution and a number of cycles in 25 years.

\[ n'(\Delta \sigma) = N_{25} \cdot \exp \left( -\frac{(\Delta \sigma)^\xi}{k} \right) \quad \text{with} \quad k = \frac{\Delta \sigma_{10^{-p}}}{\ln(10^{-p})} \quad (3) \]

Where \( n'(\Delta \sigma) \) is the number of stress cycles higher than \( \Delta \sigma \), \( N_{25} \) is the total number of stress cycles in 25 years, \( \xi \) is the shape factor of the Weibull distribution and \( k \) is the scale factor. \( 10^{-p} \) is the reference probability level where the design stress range \( \Delta \sigma_{10^{-p}} \) is defined.

The Figure 1 shows how a difference in the shape factor of the Weibull distribution can affect the fatigue damage. Because the fatigue damage depends not only on the design stress range, but on all the stress distribution, the way this distribution is approximated is important while considering the accuracy of the final result. In the current rules the design stress is defined at a probability \( 10^p \) or \( 10^{-5} \). As a consequence the total fatigue damage is very sensitive to the choice of the shape parameter. We will see that it is possible to define the design stress range at a \( 10^{-2} \) probability level, and that the overall accuracy is enhanced by cancelling the influence of the shape factor.

![Figure 1: Cumulative stress distribution (up) and stress histogram (down) defined by two Weibull distributions of different shape](image-url)
Current Rules

Three rules have been used in this study: the “Rules for the Classification of Steel Ships” from Bureau Veritas (BV rules), the “Common Structural Rules for Bulk Carrier” from the International Association of Class Societies (BC rules), and the “Common Structural Rules for Double Hull Oil Tanker” from IACS (OT rules).

All these rules are based on the same method: The stress range distribution is approximated by a Weibull distribution, and the fatigue damage is computed by combining this distribution with the appropriate S-N curve according to the Palmgren-Miner rule. These rules however differ in the choice of the reference level and the definition of the shape factor.

**BV rules**

The reference level is $10^{-5}$ and the shape factor depends on the rule length of the ship $L$ and on the vertical location of the structural detail:

\[
\xi = \xi_0 \left(1.04 - 0.14 \frac{L - T}{D - T}\right) \quad \text{without being less than} \quad 0.9 \xi_0
\]

\[
\xi_0 = \frac{73 - 0.07L}{60} \quad \text{without being less than} \quad 0.85
\]  
(4)

**BC rules**

The reference level is $10^{-4}$ and the shape factor is fixed:

\[
\xi = 1.0
\]  
(5)

**OT rules**

The reference level is $10^{-4}$ and the shape factor depends on the rule length of the ship $L$ and on the location of the structural detail:

\[
\xi = f_{\text{Weibull}} \left(1.1 - 0.35 \frac{L - 100}{300}\right)
\]

\[
0.9 < f_{\text{Weibull}} < 1.1 \quad \text{depending on the structural detail location according to Figure 2.}
\]  
(6)

![Figure 2: Distribution of \( f_{\text{Weibull}} \) factors according to OT rules](image)

Table 1: Shape parameters according to BV, BC and OT rules

<table>
<thead>
<tr>
<th>Ship length</th>
<th>100m</th>
<th>150m</th>
<th>200m</th>
<th>250m</th>
<th>314m</th>
<th>350m</th>
</tr>
</thead>
<tbody>
<tr>
<td>BV Max</td>
<td>1.14</td>
<td>1.08</td>
<td>1.02</td>
<td>0.96</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>BV Min</td>
<td>0.99</td>
<td>0.94</td>
<td>0.89</td>
<td>0.83</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>BC</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>OT Max</td>
<td>1.15</td>
<td>1.08</td>
<td>1.02</td>
<td>0.94</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>OT Min</td>
<td>0.94</td>
<td>0.89</td>
<td>0.83</td>
<td>0.77</td>
<td>0.77</td>
<td>0.73</td>
</tr>
</tbody>
</table>

The maximum and minimum values of the shape factor depending on the ship length are summarized in Table 1 for all the rules. It varies approximately in the range [0.75 ; 1.15]. The following results are based on this range for the shape factor.

Probability Level Contribution to the Fatigue Damage

In this section we are looking at purely analytical results. We will show that the stress cycles that have the most important contribution to the total fatigue damage correspond to an exceedence probability of $10^{-2}$.

**Single slope S-N curve**

We suppose that the stress range is described by a Weibull distribution, and that the SN curve is a single slope curve of slope $m$. Figure 3 shows the contribution of each probability level to the total fatigue damage. In this simplified case of a single slope S-N curve, the scale factor $k$ of the Weibull distribution and the constant $K$ of the S-N curve do not have any influence on these distributions. The only influent factors are the slope of the S-N curve and the shape factor of the Weibull distribution.

![Figure 3: Contribution of the probability levels to fatigue damage](image)

Table 2: Part of fatigue damage below the probability level $10^{-4}$

<table>
<thead>
<tr>
<th>Shape parameter of the Weibull distribution</th>
<th>0.75</th>
<th>0.9</th>
<th>1.0</th>
<th>1.15</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SN curve exponent</strong></td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.8%</td>
<td>2.6%</td>
<td>1.8%</td>
<td>1.2%</td>
</tr>
<tr>
<td>4</td>
<td>12.8%</td>
<td>6.9%</td>
<td>4.8%</td>
<td>3.0%</td>
</tr>
<tr>
<td>5</td>
<td>26.0%</td>
<td>14.7%</td>
<td>10.3%</td>
<td>6.4%</td>
</tr>
</tbody>
</table>
Another observation is that the most contributive probability level to the total fatigue damage is around 10⁻², depending on the shape factor and the slope. Table 4 shows the decimal logarithm of this most contributive level, which is the location of the maximum of the contributions shown in Figure 3. They are all between -1 and -3, with a mean value around -2. This means that even if all stress cycles between 1 and 10⁵ are contributing to fatigue damage, the biggest contributors are those corresponding to 10⁻² probability.

Table 4: Probability level of the maximum fatigue damage contribution

<table>
<thead>
<tr>
<th>Shape parameter of the Weibull distribution</th>
<th>0.75</th>
<th>0.9</th>
<th>1.0</th>
<th>1.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN curve exponent</td>
<td>3</td>
<td>1.1%</td>
<td>0.5%</td>
<td>0.3%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.6%</td>
<td>1.7%</td>
<td>1.1%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>9.3%</td>
<td>4.3%</td>
<td>2.7%</td>
</tr>
</tbody>
</table>

Double slope S-N curve

For the fatigue assessment of structural details submitted to variable cycles the state of the art is to use a double slope S-N curve. We use here a S-N curve of slope 3 and 5 with a change of slope for N=10⁷. The probability level contribution to the fatigue damage now depends on the scale factor of the Weibull distribution. In the example shown in Figure 4 the scale factor has been adjusted so that the fatigue damage reaches 1 for 10⁷, 10⁸ or 10⁹ cycles, which corresponds approximately to a fatigue life time of 2.5 years, 25 years and 250 years.

![Figure 4: Contribution of the probability levels to fatigue damage for single and double slope S-N curves (Weibull shape factor ξ = 1)](image)

In case of high damage (fatigue life corresponding to 10⁷ cycles) the results are very close to those obtained with a single slope 3 S-N curve. In case of low damage (fatigue life corresponding to 10⁹ cycles) the results are very close to those obtained with a single slope 5 S-N curve. In case of intermediate damage (fatigue life corresponding to 10⁸ cycles) the results are between those obtained with a single slope 3 and a single slope 5 S-N curve. The sharp angle observed in the dashed curve corresponding to a 10⁷ cycles fatigue life is the influence of the slope change of the SN curve. Table 5 shows the decimal logarithm of the most contributive probability level, for various shape factor of the Weibull distribution. The conclusion is the same than with a single slope S-N curve: the probability level which corresponds to the highest damage contribution is around 10⁻².

Table 5: Probability level of the maximum fatigue damage contribution for a double slope 3/5 S-N curve

<table>
<thead>
<tr>
<th>Shape parameter of the Weibull distribution</th>
<th>0.75</th>
<th>0.9</th>
<th>1.0</th>
<th>1.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cycles to damage</td>
<td>10⁷</td>
<td>-1.73</td>
<td>-1.45</td>
<td>-1.31</td>
</tr>
<tr>
<td></td>
<td>10⁸</td>
<td>-2.31</td>
<td>-1.93</td>
<td>-1.73</td>
</tr>
<tr>
<td></td>
<td>10⁹</td>
<td>-2.89</td>
<td>-2.41</td>
<td>-2.17</td>
</tr>
</tbody>
</table>

Choice of the Reference Probability Level

In order to get an accurate estimation of the fatigue life of a given structural detail, we need to have the best possible estimation of the stress cycles distribution, and especially around the 10⁻² probability area as shown in the previous section. This distribution depends on the shape factor, the design stress range and the corresponding probability, and the total number of cycles as shown in Eq (3). We will show here that if the design stress range is defined at 10⁻², the influence of the shape factor is minimized compare to the current situation where the design stress range is defined at 10⁻⁴ or 10⁻³.

Single slope S-N curve

Supposing that the real stress distribution is a Weibull distribution and that we have a perfect estimation of the design stress range at the probability 10⁻² and the number of cycles, we will compute the error in the total fatigue damage, due to a wrong estimation of the shape factor.

\[ \varepsilon = \frac{D_{app}}{D_{exact}} - 1 \]  \hspace{1cm} (7)

This error has been computed for different SN curve slopes and reference probability levels. The following figures show some results for a S-N curve with a single slope 4, and a reference level set to 10⁻² (Figure 5) or 10⁻² (Figure 6). The impact of a wrong estimation of the shape coefficient is significantly reduced when the reference probability is set to 10⁻².

In order to give the same weight to a positive error and a negative error, we define the absolute error as:

\[ \overline{\varepsilon} = \max \left( \varepsilon, \frac{1}{1+\varepsilon} - 1 \right) \]  \hspace{1cm} (8)

With this definition an error of +100% and an error of -50% have the same absolute error: 100%.

If we consider that the error on the shape factor is bounded by ±0.2, then we are interested only in the area inside the dashed grey polygon in Figure 5 and Figure 6. In this area we can look at the maximum absolute error. This error is computed for every reference level and every S-N curve slope. Results are presented in Figure 7. We see that for each S-N curve slope there is an optimum reference level where the influence of the shape factor is minimised. This optimum level is around 10⁻². Table 6 shows the maximum absolute error for the levels of reference 10⁻³, 10⁻⁴ and 10⁻².
Table 6: Maximum absolute error on fatigue damage due to a wrong estimation of the shape factor

<table>
<thead>
<tr>
<th>Reference probability level</th>
<th>10(^{-5})</th>
<th>10(^{-4})</th>
<th>10(^{-2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>m = 3</td>
<td>139%</td>
<td>98%</td>
<td>18%</td>
</tr>
<tr>
<td>m = 4</td>
<td>140%</td>
<td>87%</td>
<td>16%</td>
</tr>
<tr>
<td>m = 5</td>
<td>125%</td>
<td>67%</td>
<td>61%</td>
</tr>
</tbody>
</table>

Double slope S-N curve

In a more realistic case of a double slope 3/5 S-N curves the results are the same, and bounded by the single slope results for m=3 and m=5 (Figure 8). Results corresponding to low damage (high life time) are close to m=5 results. Results corresponding to high damage (low life time) are close to m=3 results.

Table 7 shows the maximum absolute error for the levels of reference 10\(^{-5}\), 10\(^{-4}\) and 10\(^{-2}\). The error on the total fatigue damage is considerably reduced when the reference probability level is taken at 10\(^{-2}\).

Table 7: Maximum absolute error on fatigue damage due to a wrong estimation of the shape factor, for a double slope 3/5 S-N curve

<table>
<thead>
<tr>
<th>Reference probability level</th>
<th>10(^{-5})</th>
<th>10(^{-4})</th>
<th>10(^{-2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double slope 3/5</td>
<td>150%</td>
<td>92%</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>152%</td>
<td>102%</td>
<td>11%</td>
</tr>
<tr>
<td></td>
<td>146%</td>
<td>101%</td>
<td>18%</td>
</tr>
</tbody>
</table>

The conclusion is that by choosing a 10\(^{-2}\) reference level the choice of the shape factor has hardly any influence on the estimated fatigue damage. It is therefore proposed:

- to take 10\(^{-2}\) as the reference probability level and to define the rule design load directly at this probability level,
- to take the shape factor equal to 1.0 as it is done in the BC rules (Eq 5).
Validation against Direct Computation

In this section we will verify against real stress distributions computed by direct calculations that

• a fixed shape factor associated to a $10^{-2}$ reference level gives very good approximation of fatigue damage,
• it is possible to define the design stress range directly at $10^{-2}$ using the Equivalent Design Wave concept, as it is done at $10^{-8}$ for extremes,

Direct linear quasi static hydro-structure computations have been done on a bulk carrier of rule length $L = 275.52$ m, using the procedure described by Malenica & al (2008). A partial three hold FE model (Figure 9) is coupled with the hydrodynamic model of the whole ship (Figure 10). The hydrodynamic and structural problems are solved for a speed of 15 knots and for every heading (with a 15° step). The Response Amplitude Operators of stress are computed for 186 structural details chosen on the deck, the side shell and the bottom, inside the holds, on the bulkheads, and in the inner bottom.

Figure 9: Three hold FE model of a bulk carrier with the location of the studied structural details

Figure 10: Hydrodynamic mesh and integration mesh of the bulk carrier

The linear long-term analysis is done according to IACS recommendation n°34, by combining the stress RAOs with the IACS scatter diagram describing North Atlantic wave data, using a Bretschneider wave spectrum with a $\cos^2$ spreading function, and equal probability of occurrence for all headings. The result is the distribution of stress cycles from which we can compute the stress value corresponding to the $10^{-2}$ probability, and the fatigue damage.

$10^{-2}$ reference level and fixed shape factor

What is the consequence of replacing the real stress range distribution coming from direct calculations by a Weibull distribution with a shape factor 1 passing through the exact $10^{-2}$ stress value? The fatigue damage is computed using the real stress histogram coming from the direct calculations, and using the approximated stress distribution, with a class D S-N curve defined by a double slope 3/5 and a design stress range of 53.4 MPa at $10^7$ cycles. The number of stress cycles in 25 years is the same in both computations. The two results are compared in Figure 11. Each point represents the fatigue damage of a structural detail calculated from direct and approximated calculation. We should keep in mind that:

• the same S-N curve has been used for all the details,
• the FE mesh is not a very fine mesh,
• no stress concentration factors have been applied.

Thus the absolute value of the fatigue damage is not meaningful, and we should only look at the comparison between the direct approach and the approximated approach.

Figure 11: Comparison between the fatigue damage from direct calculation and from the Weibull approximation of the stress distribution, defined at $10^{-2}$ probability with a shape factor 1

We define the error as in Eq (7) for all the 186 fatigue damage shown in Figure 11, and the error range corresponding to a 80% confidence interval: In this case, for 80% of the details, the error from the Weibull approximation is within the range [-6% ; -3%].

In order to compare with the current situation, the damage is also computed using the BV, BC and OT rule methods: the stress histogram is approximated by a Weibull distribution based on the $10^{-5}$ or $10^{-4}$ stress range and a shape factor defined in equations (4) (5) and (6). The stress value at the reference probability and the number of stress cycles are taken from the direct calculations. The differences shown in Figure 12 is then only due to the Weibull approximation. Considering the length of the studied ship and the location of the structural details, the shape factors used are in the range [0.81 ; 0.98] for the OT rule and [0.81 ; 0.93] for the BV rule. Figure 13 shows the cumulative distribution of the error. The 80% confidence error ranges corresponding to each approximation are shown in Figure 13 and given in Table 8. These results confirm that the proposed method (Weibull distribution based on the $10^{-2}$ stress and a shape 1) gives a very good approximation of the fatigue damage, and that the errors are considerably reduced compare to the existing rule methods.
We begin with the direct calculations and then use the Weibull distribution to approximate the stress distribution, employing BV, BC, and OT rules.

**Figure 12:** Comparison between the fatigue damage from direct calculation and from the Weibull approximation of the stress distribution, using BV, BC and OT rules.

We then show the cumulative distribution of the error on the 25 years fatigue damage computed with the Weibull approximation, using BV, BC and OT rules, and the new approach at $10^{-2}$ probability level.

**Figure 13:** Cumulative distribution of the error on the 25 years fatigue damage computed with the Weibull approximation, using BV, BC and OT rules, and the new approach at $10^{-2}$ probability level.

**Table 8:** Error range on the fatigue damage due to the Weibull approximation.

<table>
<thead>
<tr>
<th>Method</th>
<th>Reference probability</th>
<th>Shape factor</th>
<th>Error range</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>$10^{-2}$</td>
<td>1</td>
<td>[-6% ; -3%]</td>
</tr>
<tr>
<td>BC rules</td>
<td>$10^{-4}$</td>
<td>1</td>
<td>[-4% ; 34%]</td>
</tr>
<tr>
<td>OT rules</td>
<td>$10^{-4}$</td>
<td>[0.81 ; 0.98]</td>
<td>[-40% ; 28%]</td>
</tr>
<tr>
<td>BV rules</td>
<td>$10^{-5}$</td>
<td>[0.81 ; 0.93]</td>
<td>[-56% ; -23%]</td>
</tr>
</tbody>
</table>

**Design Waves for the $10^{-2}$ loads**

In order to define the stress distribution by a Weibull curve based on the $10^{-2}$ stress, we should be able to define this $10^{-2}$ design stress with the same accuracy as the $10^{-4}$ or $10^{-5}$ design stress defined in the current rules. For practical reasons the $10^{-2}$ stress has to be computed with a limited number of load cases. The method used is the Equivalent Design Wave method, which is already used to define the extreme load cases and the fatigue load cases (see Bureau Veritas rules, Pt B, Ch 7, App 3). This method is briefly explained here.

1. A limited number of dominant loads are defined. Here we defined 5 dominant loads:
   - Vertical acceleration at fore perpendicular
   - Vertical bending moment amidship
   - Roll
   - Pressure at waterline amidship
   - Torsional moment at 0.25 L

2. For each of those five dominant loads the $10^{-2}$ value is computed from the direct long-term analysis, and a regular design wave is defined: this is a wave that creates the $10^{-2}$ design load. The frequency and heading of the wave are chosen according to the maximum of the corresponding RAO, and the wave height is adjusted to reach the design load value. The stresses in all the structural details are computed for each of the five design waves.

3. For each detail the maximum stress among the five design waves is taken as the approximated $10^{-2}$ stress. This stress is then compared to the exact $10^{-2}$ stress coming from the long-term analysis.

The same procedure is applied for the $10^{-4}$, $10^{-5}$ and $10^{-8}$ stress. The design waves used here are the same than for $10^{-2}$, except for the $10^{-8}$ stress where two design waves are used for the vertical bending moment: one in head sea and another one in following sea. The results are shown in Figure 14 and Figure 15 (the results for $10^{-5}$ are not shown but are very close to those for $10^{-4}$).

**Figure 14:** Comparison between the $10^{-2}$ and $10^{-4}$ stress range from direct calculation and Equivalent Design Wave approach.
CONCLUSIONS

The usual rule approach to compute fatigue damage is to approximate the stress range distribution by a Weibull distribution based on a shape factor, a design value at a given probability level and a total number of cycles. We have shown that for usual S-N curves and usual range of shape factor, the most important contribution to fatigue damage is coming from the stress ranges corresponding to a probability around $10^{-2}$.

A sensitivity analysis has been done on the choice of the reference level to define the stress distribution. If the design stress is directly defined at $10^2$, the influence of the shape factor on the total damage is nearly cancelled, whereas the shape factor is very important if the design stress is defined at $10^{-7}$ or $10^{-5}$. It is then proposed to define the fatigue design stress at this probability level and to fix the shape factor to 1, as it is done in the BC rules.

By comparisons with direct computations, it has been shown that these assumptions are very good, and that it is possible to define the $10^2$ stress with the Equivalent Design Wave method, as it is done for the $10^{-7}$, $10^{-5}$ or $10^{-8}$ stress, with the same level of accuracy. The error made in the evaluation of the design stress range is approximately the same at all probability levels. However the influence of the Weibull approximation is considerably reduced when the design stress is defined at $10^{-2}$ probability.

The proposed methodology is currently being applied in the IACS Harmonized Common Structural Rules under development. The next step for the BV rule elaboration is to adapt the rule formula to the loads at $10^{-2}$ probability level for fatigue damage verification based on the corresponding rule load cases.

REFERENCES

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Bureau Veritas, 2010. “Common Structural Rules for Bulk Carrier”