Analytical solutions to diffraction of concentric porous cylinder system with cylinder hull of varying porosity

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Abstract

Wave diffraction of concentric porous cylinder system with cylinder hull of varying porosity is studied by using analytical method based on eigenfunction matching. The fluid domain around the structure hull are divided into three subdomains and in each of subdomain, an eigenfunction expansion of the velocity potential is obtained by satisfying the Laplace equation and the boundary conditions on the freesurface, on the seabed. The unknown coefficients of eigenfunction expansions are determined by derivation continuity condition and porous theory on the cylinder hull. The porous theory present in this paper comes form Chwang *et al.*(1983), which is based on the assumption that the flow in the porous medium is governed by Darcy's law. Two porous-effect parameters which exist in two porous cylinder are the functions of z instead of a constant number. It is not only contain porous cylinder hull of uniform but also the porous cylinder hull of varying porosity.

Keywords: Diffraction , eigenfunction, varying porosity, porous-effect parameter function, wave force

1 Introduction

A vertical cylindrical porous structure of circle section is not only regard as an academic case in hydrodynamic analysis due to its simple geometric form, but also its wide and important applications in various engineering aspects. The porous structure can be frequently constructed to protect coastal and offshore structures against direct wave force.

Interaction of wave force with a solid vertical cylinder have been an active research topic for years. It has been started with MacCamy & Fuch *et al.*(1954) on the simplest case of a vertical cylinder standing on the seabed. Since Miles and Gilbert *et al.*(1968) formulated the problem of the scattering of the surface waves by a circular dock and obtained a variational approximation to the far filed, a number of works have been done for different types of vertical cylinders (truncated and composed of different diameters) by using so called eigenfunction matching method which consists of dividing the fluid domain into several cylindrical subdomains and matching the eigenfunction expansions in different subdomain. More recently by Liu *et al.*(2012) for a series of different configurations of a single cylinder.

In recent years , many researchers have studied the wave interaction with porous structures through the experiment and numerical. Chwang *et al.*(1983) studied the porous-wavemaker theory which is based on the assumption that the flow in the porous medium is governed by Darcy's law. He present that the normal velocity of the fluid passing through the porous cylinder is thus linearly proportional to the pressure difference between the two sides of the outer porous cylinder. Following his idear, TengBin *et al.*(2000) and Zhong *et al.*(2006) have reported the case of Two Uniform Columns and porous outer wall by employing Hankel functions for the solutions of velocity potential in the inner domain and outter domain. Then not limited attention to wave diffraction by a concentric two cylinder system, CaiZeWei *et al.*(2002) have done only one part-porous cylinder standing on the seabed by using the analytical method. But in his formulae, he has ignored the evanescent modes which is important for numerical results. Very Recently , Zhu *et al.*(2011) describes a plane regular wave interaction with a combined cylinder which consists of a solid inner column and a coaxial perforated outer cylinder, and the non-linear boundary condition at the perforated wall is in the study. Liu *et al.*(2012) using the boundary element method based on the linear wave theory, studied the concentric dual cylindrical system. Variance with other work, Molin *et al.*(2011) present a quadratic , not linear ,law, relating the pressure differential to the traversing velocity. His model the openings are large and sharp enough for the flow to separate and frictional forces to be negligible as compared to pressure force.

The case present in this paper, concentric porous cylinder system with cylinder hull of varying porosity, have contain lots of condition done before and shown in Figure 1. Two porous-effect parameters which exist in two porous cylinder are the functions of z instead of a constant number. The porous-effect parameters $G/\epsilon(z)$ can be not only a piecewise function but also a continuous function. In this paper, there are three kinds of porous-effect parameter function : a piecewise function; a continuous function with constant number; a continuous function similar with incident wave such as :$G(z) = \cosh k_0 h \cosh k_0(z+h)/(2k_0h+\sinh 2k_0h)$.

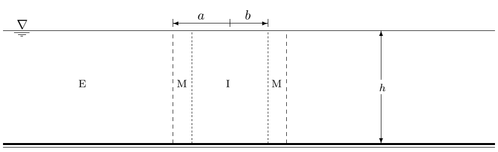

Figure 1: concentric porous cylinder system with cylinder hull of varying porosity

2 Mathematical formulations

We defined the cylindrical coordinated system (r, θ, z) by its origin located at the center of the cylinder and on the mean plane of the free surface for each case. The axis oz is vertically upward. By assuming that the fluid is perfect and flow irrotational, fluid velocity $\vec{V}(r, \theta, z, t)$ can be represented by the gradient of a potential function $\Phi(r, \theta, z, t)$ which satisfies the Laplace's equation:

$$\vec{V}(r,\theta,z,t) = \nabla\Phi(r,\theta,z,t) \quad \text{then} \quad \nabla^2\Phi(r,\theta,z,t) = 0 \tag{1}$$

in the fluid domain. Under the assumption of infinitesimal waves, we consider only the linear potential proportional to the wave steepness. Furthermore, an analysis in frequency domain is performed so that the factor $\exp(-i\omega t)$ representing the time-harmonic variation will be omitted in the following for the sake of simplicity.

On the mean free surface ($z = 0$), the combined kinematic and dynamic boundary condition is given by:

$$-\nu\Phi + \partial_z\Phi = 0 \tag{2}$$

in which $\nu = \omega^2/g$ with ω as wave frequency and g the acceleration due to gravity.

On the cylinder hull is expressed by:

$$\partial_n\Phi = V_n \tag{3}$$

where V_n is the cylinder's velocity in the direction normal to the hull. For the diffraction problem , we have $V_n = 0$. In the same way, the boundary condition on the sea bed ($z = -h$) is $\partial_z\Phi = 0$. In addition, we have to a radiation condition requiring that all perturbation due to cylinders disappears at infinity, i.e.

$$\Phi \to \Phi_0 \qquad \text{at} \qquad r \to \infty \tag{4}$$

to ensure the uniqueness of the solution. In (4), Φ_0 representing the velocity of incoming waves is given as:

$$\Phi_0 = -\frac{Ag}{\omega}\frac{\cosh k_0(z+h)}{\cosh k_0 h}\exp(ik_0 r\cos\theta) \tag{5}$$

in which (A, ω, k_0) stand for the amplitude, frequency and wavenumber, respectively, of incoming waves propagating along the positive $x-$ axis.

2.1 Eigenfunction expressions in a cylindrical domain

To satisfy the Laplace equation, the velocity potential should be a combination of eigenfunctions which can be expressed as:

$$\begin{aligned}\Phi(r,\theta,z) &= \{\log(r)\} + \{r^\ell, r^{-\ell}\}\{\cos\ell\theta, \sin\ell\theta\}\\ &+\{\cos k_n z, \sin k_n z\}\{\mathbf{I}_\ell(k_n r), \mathbf{K}_\ell(k_n r)\}\{\cos\ell\theta, \sin\ell\theta\}\\ &+\{\cosh k_0 z, \sinh k_0 z\}\{\mathbf{J}_\ell(k_0 r), \mathbf{Y}_\ell(k_0 r)\}\{\cos\ell\theta, \sin\ell\theta\}\end{aligned} \tag{6}$$

in which $(\mathbf{J}_\ell, \mathbf{Y}_\ell)$ are defined in Abramowitz & Stegun (1967), as the ℓth order Bessel functions of the (first,second) kind, respectively, while $(\mathbf{I}_\ell, \mathbf{K}_\ell)$ the ℓth order modified Bessel functions of the first and second kinds, respectively.

Indeed, the potential of incoming waves can be expanded by developing $\exp(ik_0 r\cos\theta)$ into series so that:

$$\Phi_0 = -\frac{Ag}{\omega}\sum_{\ell=0}^{\infty}\phi_0^\ell\cos\ell\theta \tag{7}$$

with:

$$\phi_0^\ell = \epsilon_0^\ell Z_0(k_0, z, h)\mathbf{J}_\ell(k_0 r) \tag{8}$$

in which:

$$Z_0(k_0, z, h) = \cosh k_0 h\cosh k_0(z+h)/(2k_0 h + \sinh 2k_0 h) \tag{9}$$

and the unknown coefficients:

$$\epsilon_0^\ell = \begin{cases} 1/Z_0(k_0, 0, h) & \text{for} \quad \ell = 0\\ 2i^\ell//Z_0(k_0, 0, h) & \text{for} \quad \ell \geq 1\end{cases}$$

The wave number k_0 is defined by the dispersion equation $k_0 \tanh k_0 h = \nu$ obtained by satisfy the boundary condition at the free surface.

In the same way the velocity potential Φ of wave diffraction around cylindrical structures can be expressed as:

$$\Phi(r, \theta, z) = -\frac{Ag}{\omega} \sum_{\ell=0}^{\infty} \phi^{\ell}(r, z) \cos \ell\theta \qquad (10)$$

The appropriate expression of $\phi^{\ell}(r, z)$ composed of eigenfunctions should be obtained by considering the boundary conditions in a cylindrical domain of fluid.

2.2 Porous cylinder theory

boundary condition on the porous cylinder can be written as:

$$\partial_r \Phi = V(\theta, z) \quad (r = a, 0 \le \theta \le 2\pi, -t \le z \le 0) \qquad (11)$$

using Bernoulli equation, Dynamic water pressure can be :

$$P = \rho \cdot i\omega\Phi \qquad (12)$$

Following Chwang *et al.*(1983), the normal velocity of the fluid passing through the porous cylinder is thus linearly proportional to the pressure difference between the two sides of the outer porous cylinder:

$$V(\theta, z) = \frac{\gamma}{\mu}(P^+ - P^-) \qquad (13)$$

where γ is the material constant and μ is the dynamic viscosity. using the dimensionless $\epsilon = \mu k_0/\gamma\rho\omega$, the relationship can be written as:

$$\epsilon \partial_r \Phi - ik_0(\Phi^+ - \Phi^-) = 0 \qquad (14)$$

where: k_0 is the wave number $\epsilon = 0$ stands for $\phi^+ = \phi^-$, the full transparent condition and $\epsilon = \infty$ stands for $\partial_r \phi = 0$, the solid boundary condition.

Porous-effect parameters which exist in two porous cylinder are the functions of z instead of a constant number. If $\epsilon(z)$ is a piecewise function, can be written as:

$$\epsilon(z) = \begin{cases} \epsilon & (-h \le z \le t - h) \\ 0 & (t - h \le z \le 0) \end{cases} \qquad (15)$$

if $\epsilon(z)$ is a continuous function, and a constant number, can be written as:

$$\epsilon(z) = \epsilon \qquad (-h \le z \le 0) \qquad (16)$$

if $\epsilon(z)$ is a continuous function, and similar function with incident wave, can be written as:

$$\epsilon(z) = \epsilon \frac{\cosh k_0 h \cosh k_0(z + h)}{(2k_0 h + \sinh 2k_0 h)} \quad (-h \le z \le 0) \qquad (17)$$

2.3 Potential expansions in different domain

As illustrated on Figure 1, we denote the inner cylinder radius by b and the outer cylinder radius by a and the waterdepth is denoted by h. The fluid domain outside of the extended porous cylinder of the exterior radius ($r > a$) is denoted by E while the fluid domain inside the porous cylinder outside the solid cylinder ($b < r < a$) is denoted by I .

Following Garrett (1970), the velocity potential in the exterior domain $E(r \ge a)$ is written as:

$$\phi_E^{\ell} = a_0^{\ell} Z_0(k_0, z, h) \frac{\mathbf{H}_{\ell}(k_0 r)}{\mathbf{H}'_{\ell}(k_0 a)} + \sum_{n=1}^{\infty} a_n^{\ell} Z_n(k_n, z, h) \frac{\mathbf{K}_{\ell}(k_n r)}{\mathbf{K}'_{\ell}(k_n a)} + \phi_0^{\ell} \qquad (18)$$

with:

$$Z_n(k_n, z, h) = \cos k_n h \cos k_n(z + h)/(2k_n h + \sin 2k_n h) \qquad (19)$$

and k_n defined by $k_n \tan k_n h = -\nu$ for $n \ge 1$. The function $Z_0(k_0, z, h)$ are defined by (8) and $Z_n(k_n, z, h)$ by (19) for $n \ge 1$ ensure that ϕ_E^{ℓ} satisfy the boundary condition $(-\nu + \partial_z)\phi_E^{\ell} = 0$ at the mean free surface ($z = 0$) and $\partial_z \phi_E^{\ell}$ at the sea bed ($z = -h$).

In the domain $M(b \le r \le a)$ inside the porous cylinder, the velocity potential is written as:

$$\phi_M^{\ell} = b_0^{\ell} Z_0(k_0, z, h) \frac{\mathbf{J}_{\ell}(k_0 r)}{\mathbf{J}'_{\ell}(k_0 a)} + \sum_{n=1}^{\infty} b_n^{\ell} Z_n(k_n, z, h) \frac{\mathbf{I}_{\ell}(k_n r)}{\mathbf{I}'_{\ell}(k_n a)} + c_0^{\ell} Z_0(k_0, z, h) \frac{\mathbf{H}_{\ell}(k_0 r)}{\mathbf{H}'_{\ell}(k_0 a)} + \sum_{n=1}^{\infty} c_n^{\ell} Z_n(k_n, z, h) \frac{\mathbf{K}_{\ell}(k_n r)}{\mathbf{K}'_{\ell}(k_n a)} \qquad (20)$$

with the same function $Z_n(k_n, z, h)$ and wavenumber k_n for $n \ge 0$ as those for ϕ_E^{ℓ} since ϕ_T^{ℓ} satisfy the same boundary condition as ϕ_E^{ℓ} at the mean free surface and the sea bed.

In the domain $I(b \le r \le a)$ inside the porous cylinder, the velocity potential is written as:

$$\phi_M^{\ell} = d_0^{\ell} Z_0(k_0, z, h) \frac{\mathbf{J}_{\ell}(k_0 r)}{\mathbf{J}'_{\ell}(k_0 a)} + \sum_{n=1}^{\infty} d_n^{\ell} Z_n(k_n, z, h) \frac{\mathbf{I}_{\ell}(k_n r)}{\mathbf{I}'_{\ell}(k_n a)} \qquad (21)$$

with the same function $Z_n(k_n, z, h)$ and wavenumber k_n for $n \ge 0$ as those for ϕ_E^{ℓ} since ϕ_T^{ℓ} satisfy the same boundary condition as ϕ_E^{ℓ} at the mean free surface and the sea bed.

2.4 Matching equation

The coefficients $a_n^\ell, b_n^\ell, c_n^\ell, d_n^\ell$ for $0 \le (\ell, n) \le \infty$, are unknown to be determined by the derivation continuity condition and porous theory:

$$\int_{-h}^{0} \partial_r \phi_I^\ell|_{(r=b)} \cosh k_0(z+h)\mathbf{d}z = \int_{-h}^{0} \partial_r \phi_M^\ell|_{(r=b)} \cosh k_0(z+h)\mathbf{d}z \quad \text{(22a)}$$

$$\int_{-h}^{0} \partial_r \phi_I^\ell|_{(r=a)} \cos k_n(z+h)\mathbf{d}z = \int_{-h}^{0} \partial_r \phi_M^\ell|_{(r=a)} \cos k_n(z+h)\mathbf{d}z \quad \text{(22b)}$$

$$\int_{-h}^{0} ik_0\epsilon_I(z)(\phi_I^\ell - \phi_M^\ell)|_{(r=b)} \cosh k_0(z+h)\mathbf{d}z = \int_{-h}^{0} \partial_r \phi_M^\ell|_{(r=b)} \cosh k_0(z+h)\mathbf{d}z \quad \text{(22c)}$$

$$\int_{-h}^{0} ik_0\epsilon_I(z)(\phi_I^\ell - \phi_M^\ell)|_{(r=b)} \cos k_n(z+h)\mathbf{d}z = \int_{-h}^{0} \partial_r \phi_I^\ell|_{(r=b)} \cos k_n(z+h)\mathbf{d}z \quad \text{(22d)}$$

$$\int_{-h}^{0} \partial_r \phi_M^\ell|_{(r=a)} \cosh k_0(z+h)\mathbf{d}z = \int_{-h}^{0} \partial_r \phi_E^\ell|_{(r=a)} \cosh k_0(z+h)\mathbf{d}z \quad \text{(22e)}$$

$$\int_{-h}^{0} \partial_r \phi_M^\ell|_{(r=a)} \cos k_n(z+h)\mathbf{d}z = \int_{-h}^{0} \partial_r \phi_E^\ell|_{(r=a)} \cos k_n(z+h)\mathbf{d}z \quad \text{(22f)}$$

$$\int_{-h}^{0} ik_0\epsilon_E(z)(\phi_M^\ell - \phi_E^\ell)|_{(r=a)} \cosh k_0(z+h)\mathbf{d}z = \int_{-h}^{0} \partial_r \phi_M^\ell|_{(r=a)} \cosh k_0(z+h)\mathbf{d}z \quad \text{(22g)}$$

$$\int_{-h}^{0} ik_0\epsilon_E(z)(\phi_M^\ell - \phi_E^\ell)|_{(r=a)} \cos k_n(z+h)\mathbf{d}z = \int_{-h}^{0} \partial_r \phi_M^\ell|_{(r=a)} \cos k_n(z+h)\mathbf{d}z \quad \text{(22h)}$$

The equation (22a) and (22b) are the Galerkin form of derivation continuity condition on the inner cylindrical surface. The equation (22c) and (22d) are the Galerkin form of porous condition on the inner changed-porous surface. The equation (22e) and (22f) are the Galerkin form of derivation continuity condition on the outer cylindrical surface. The equation (22g) and (22h) are the Galerkin form of porous condition on the outer changed-porous surface. The equation (22) give a linear system to determine (a_0^ℓ, a_n^ℓ) and (b_0^ℓ, b_n^ℓ), (c_0^ℓ, c_n^ℓ) and (d_0^ℓ, d_n^ℓ) for $1 \le n \le N$ with N the truncated number of the infinite series in potential.

2.5 Wave force

After having obtained the analytical solution of velocity potentials in different domains, we evaluate the wave forces by the direct integration of the hydrodynamic pressure on two porous cylinder hull of varying porosity. The linear wave force on outer porous cylinder along the x-axis F_{xo} is given by:

$$F_{xo} = i\pi\rho g A \left\{ a \int_{-h}^{0} [\phi_E^1(a,z) - \phi_M^1(a,z)]\mathrm{d}z \right\} \quad (23)$$

The linear wave force on the inner porous cylinder along the x-axis F_{xi} is given by:

$$F_{xi} = i\pi\rho g A \left\{ b \int_{-h}^{0} [\phi_M^1(b,z) - \phi_I^1(b,z)]\mathrm{d}z \right\} \quad (24)$$

3 Numerical Results

In this part, First we consider the inner cylinder is a solid cylinder for $\epsilon_i = \infty$. Porous-effect parameter in outer cylinder is a piecewise function. In order to compare with TengBin *et al.*(2000), the formulae $t = h$. Secondly, we consider the two-porous cylinder the porous-effect parameter in inner cylinder is varying.

3.1 Inner cylinder is a solid cylinder $\epsilon_i = \infty$

In this part shows wave force on inner cylinder and outer cylinder for inner cylinder is a solid

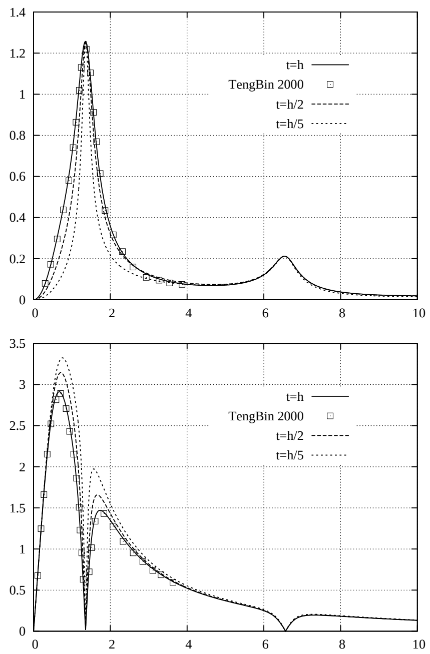

Figure 2: wave force for $\epsilon_i = \infty$ and $\epsilon_e = 5$

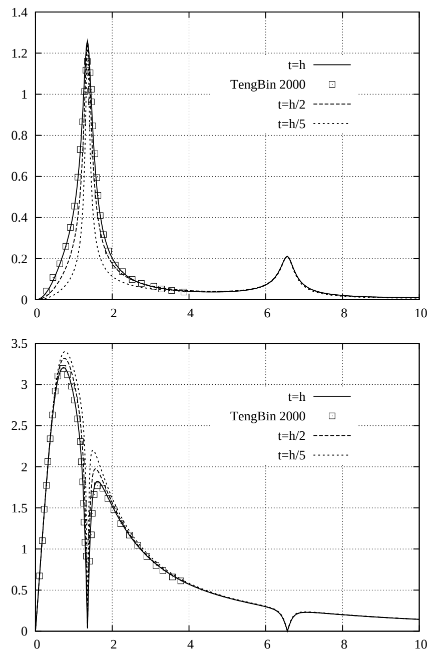

Figure 3: wave force for $\epsilon_i = \infty$ and $\epsilon_e = 10$

cylinder $\epsilon_i = \infty$. The condition is $b = a/2; h = a$ and Figure 2,3,4 shows $\epsilon = 5, 10, 20$ respectively.

3.2 Inner cylinder is a porous cylinder

In this part, wave force on inner cylinder and outer cylinder for outer and inner cylinder is a porous cylinder. The condition is $b = a/2; h = a$ and Figure 5,6,7 shows $\epsilon_e = 5, 10, 20$ respectively.

4 Conclusions

The diffraction of concentric porous cylinder system with hull of varying porosity is analyzed, for the first time to authors' knowledge, by using the semi-analytical method based on eigenfunction expansions of velocity potentials in different subdomains and porous-theory by Chwang *et al.*(1983) however modified idea for hull of varying porosity and two porous hull. The porous-effect parameter is the function $G(z)$ instead of a constant number. From the results , with the outer cylinder effect the wave force on the inner cylinder is reduced and with the inner cylinder effect the wave force on the outer cylinder is also reduced.

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References

[1] Teng Bin , Han Lin & Li Yucheng(2000) Wave Diffraction from a vertical cylinder with two uniform columns and porous outer wall. *China Ocean Engineering*vol.14 No.3 pp297-306

[2] 蔡泽伟，叶春阳，宋晓刚 (2002) 部分可渗透直立圆柱的消波作用研究　水动力学研究与进展 vol.17 No.3

[3] Taylor(1956) Fluid Flow in Regions Bounded by Porous Surface *Proc.R.Soc.Lond.*A234,456-475

[4] Taylor(1956) Fluid Flow in Regions Bounded by Porous Surface *Proc.R.Soc.Lond.*A234,456-475

[5] Chwang(1983) A porous-wavemaker theory *J . Fluid Mech*vol.132, pp395-460

[6] Xiping Yu and Chwang(1994) Wave - induced oscillation in Harbor with porous breakwaters *Ocean Eng*125-144

[7] MacCamy R.C. & Fuchs R.A.(1954) Wave forces on piles : a diffraction theory. *US Army Corps of Engineers Beach Erosion Board.*Tech.Memo.No.69.

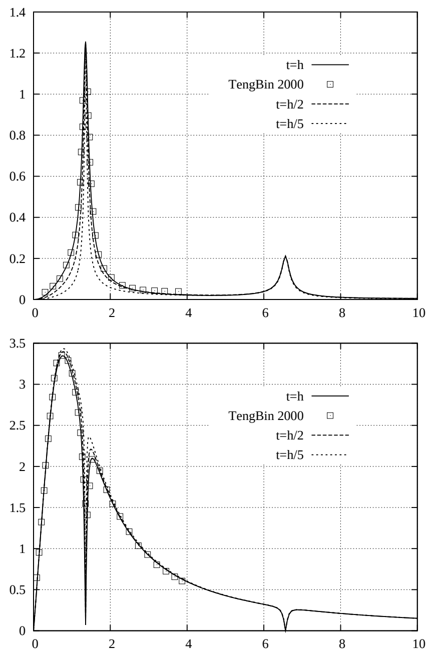

Figure 4: wave force for $\epsilon_i = \infty$ and $\epsilon_e = 20$

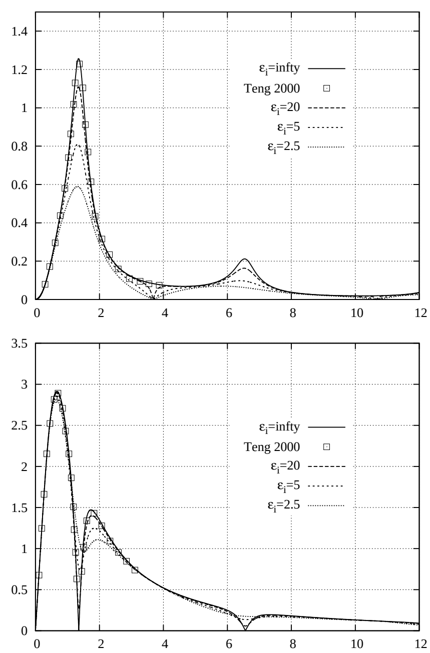

Figure 5: wave force for $\epsilon_e = 5$ inner cylinder is a porous cylinder

[8] Miles,J.W. & Gilbert,J.F.(1968) Scattering of gravity waves by a circular dock. *J.Fluid Mech.* 34,783.

[9] Garrett C.J.R.(1971) Waves forces on a circular dock *J.Fluid Mech.* 46, 129-39.

[10] Yeung R.(1981) Added mass and damping of a vertical cylinder in finite-depth waters. *Appl. Ocean res* . 3(3),119-33

[11] Chen X.B(1991) Analytical evaluation of the second-order loads upon a tension leg platform column *Institut Francais du Petrole*, Ref. IFP No.38 739. Rueil-Malmaison (France)

[12] M.K.M.Darwiche, A.N.Williams and K-H.Wang(1994) Wave interaction with semi-porous cylindrical breakwater *Ocean Engineering*, 1994.120.382-403

[13] A.N.Williams (1997) Wave interaction with a semi-porous cylindrical breakwater mounted on a storage tank *Ocean Engineering*, vol.25,pp.195-219

[14] A.N.Williams, W.Li, K.-H. Wang (1998) Wave wave interaction with a floating porous cylinder *Ocean Engineering*, 21 October 1998

[15] H. X. Liu, W. Y. Duan, and X.B. Chen (2012) Analytical solution of wave loads on cylindrical strctures. *Proc. ICMT2012,Harbin (China)*

[16] Molin (2011) Hydrodynamic modeling of perforated structures *Applied Ocean Research*33(2011)1-11

[17] Williams & Li(2000) Water wave interaction with an array of bottom-mounted surface-piercing porous cylinders *Ocean Engineering*27(2000)841-866

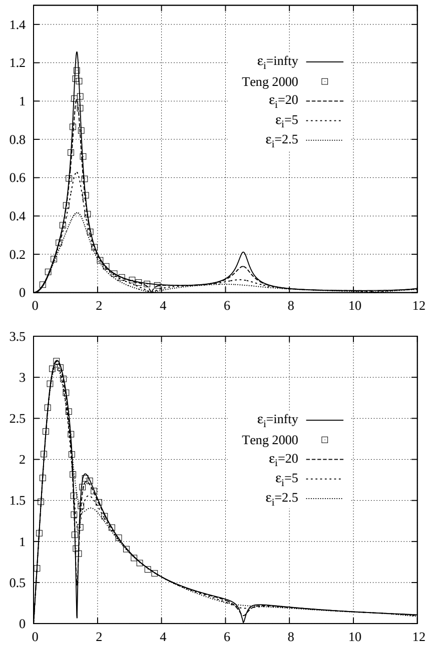

Figure 6: wave force for $\epsilon_e = 10$ inner cylinder is a porous cylinder

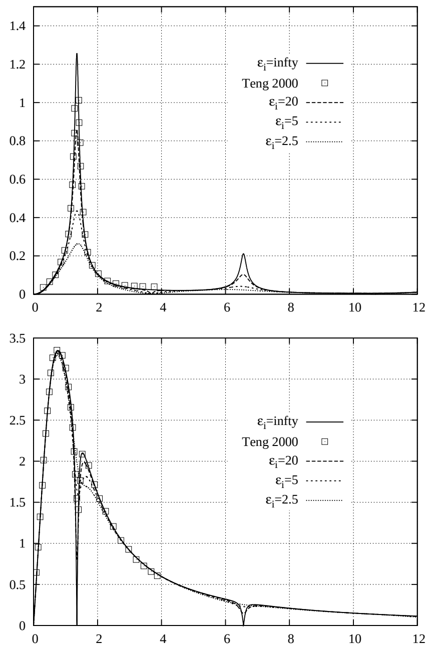

Figure 7: wave force for $\epsilon_e = 20$ inner cylinder is a porous cylinder