Simulation of Ice Loads on Ship Hull

Vladimir Tryaskin and Vladimir Yakimov
Ship Structure Department, State Marine Technical University
Saint-Petersburg, Russian Federation

Pierre Besse
Research Department, Bureau Veritas – Marine Division
Neuilly-sur-Seine, France

ABSTRACT

The paper covers the statement of simulation problem for ice loads acting on ship hull. The application of probabilistic approach to ice loads’ estimation is substantiated. The main theoretical propositions as well as the general algorithm and the basic scheme of simulation process implementation are adduced. A number of test examples are considered relating to the hull of Samotlor-type ice-going oil tanker. The obtained results are analyzed. The special-purpose software developed by the authors is used for probabilistic computer experimentation.

KEY WORDS: Simulation; ice loads; ship hull; hydrodynamic impact model; probabilistic computer test; pseudo-random number generators; distributions of stochastic quantities.

INTRODUCTION

The ship hull interaction with ice is of random nature, therefore it is reasonable to apply probabilistic rather than deterministic method for estimating the ice loads acting on the ship hull. The modern hardware and software tools are able to simulate complicated processes and investigate the same by performing computer experiments taking into account various random factors.

The use of simulation (statistical modeling) techniques for estimation of ice loads make it possible to formulate and solve a number of important practical problems such as:
– estimation the standard probability of occurrence for design ice loads on ice-strengthened hull structures;
– designing the hull structure of ice-class ships for the specified probability of design ice load occurrence;
– estimation the equivalent strength for various ships relating to the same ice category or for various structures of the same ship taking into account ice load variations;
– prediction the damage rate of ship hull structures (or risk assessment) under various in-service ice conditions;
– justification the ice category assigned to a ship in accordance with the actual ice conditions.

In addition, the simulation of ice loads allows us to account for their probabilistic nature in the development of special ship documentation and on-board software used by ship operators to determine safe operating modes in ice (primarily, the allowable ship speed). The implementation of this approach would enhance the ship safety in ice.

The research into ice loads’ simulation was initiated at the Ship Structure Department of St. Petersburg State Marine Technical University in the second half of the 1980s and these efforts were primarily associated with Kurdyumov. He formulated a general statement of the problem, set up the theoretical basis for the random ice load simulation technique based on the hydrodynamic model of solid body impact on ice and put forward the principles for design and performance of the probabilistic experiment. In 1992 all active research efforts in this field were suspended. However, this problem remained to be of interest in the course of time, and in 2008 the research in this field was revitalized.

The method suggested here for simulation of ice loads on ship hull is based on a combination of a simulation algorithm and generators of pseudo-random numbers. The problem in its initial formulation is a statistical one. The goal of the method is to justifiably select the theoretical distribution laws for random ice load parameters and subsequently estimate the average values of the parameters under consideration. These empirical distribution functions (bar charts) are plotted based on the results of a probabilistic computer experiment.

The characteristic feature of ship hull/ice interaction process is a significant number of various stochastic factors directly influencing the ice load magnitude. Under these conditions it is practically impossible to identify the individual contributions of specific factors to the final parameters of ice loads or to assess the correlations between these factors. The process of ship operation in ice can be represented as a series of its operations on certain shipping routes. Each shipping route is subdivided into zones based on the peculiarities of sea areas. The boundaries of these zones are primarily set based on the spatial and temporal variability of ice conditions. The ice conditions in adjacent zones are expected to be significantly different and distinguished by specific features. Therefore, an integral part of the probabilistic model is a simulation model of random ice loads on ship hull at the specified shipping route.

The process of ice load simulation modeling can be represented as a
series of steps:
– preparation of initial data for the ship and the geographical area of its operation in ice;
– simulation of ice conditions (ice thickness, linear size, concentration) at the specified shipping zone/route using the bar charts of partial distributions;
– simulation of ice physical and mechanical properties (temperature, salinity, density, ultimate bending strength, ultimate crushing strength, conditional parameter of dynamic ice crushing strength, Young's modulus, Poisson's constant) at the specified shipping zone/route;
– simulation of ship operating mode in ice (continuous motion or ramming; operation in solid or broken ice of different concentration) and acceptable ship speed in ice at the specified shipping zone/route;
– simulation of impact impulse distribution over the ship hull length at the actual waterline level;
– estimation of ice load parameters (load intensity, linear load, total contact force, linear dimensions of ship hull/ice contact zone, depth of hull side penetration into ice, etc.) using the hydrodynamic solid body/ice impact model suggested in the mid-1970s (Kurdyumov and Kheisin, 1976), generalized by Tryaskin on the case of impact against the ice, destroyed due to bending (Kurdyumov, Tryaskin and Kheisin, 1979) and used widely in Russia in the following years applied to ice-going ships and icebreakers;
– accumulation, classification and processing of the obtained results using the mathematical statistic processing techniques.

MAIN PROPOSITIONS

Modeling of Ice Conditions’ Parameters Using Bar Charts of Partial Distributions

The modern surveillance and data acquisition systems are able to take quick high-precision automatic measurements of ice cover thickness (radars, telemetry systems, etc.), its linear dimension and concentration (digital aerial survey, satellites, etc.). Since the ice cover thickness is governing the ice resistance and ice load magnitude, the ice thickness data should be used as the initial data input in the simulation process under consideration.

The values of ice cover thickness obtained by ice survey are used to plot the partial distributions’ bar chart which defines the probability of ice cover with a fixed thickness gradation at the specified shipping zone/route. Analytically, the partial distributions can be represented by three number groups: group of left-hand (lower) limits of ice thickness ranges; group of right-hand (upper) limits of ice thickness ranges and group of ordinates corresponding to the probability that the measured thickness values would fall within the specified range. In this study seven ice thickness ranges are considered: 0.01 - 0.10 m, 0.10 - 0.30 m, 0.30 - 0.70 m, 0.70 - 1.20 m, 1.20 - 1.80 m, 1.80 - 2.70 m and 2.70 - 5.00 m, which is in full compliance with the generally accepted classification of ice cover by age (nilas, young ice, thin first-year ice, medium first-year ice, thick first-year ice, second-year ice and multi-year ice).

A random value (realization) of ice thickness is determined directly from the partial distributions’ chart plotted for the specified shipping zone/route. If the thickness values within each range are assumed to be distributed uniformly, then the realization of ice cover thickness will be obtained by the following steps:
– a random value $\xi$ is generated (using a standard generator of pseudo-random values) which is uniformly distributed in the interval $(0, 1)$;
– the design range of thickness is found to be valid for the two-sided inequality $\Phi_{j-1} < \xi \leq \Phi_j$, where $\Phi_j = \sum_{i=0}^{j} P_i$, note that $\Phi_0 = 0$ and $\Phi_N = 1$ ($P_i$ – probability of falling within the thickness range; $N$ – assumed number of thickness ranges);
– random value of ice thickness cover is calculated from $H = H_{2i} - (H_{2i} - H_{1i}) \frac{\xi - \xi_{i-1}}{\xi_i - \xi_{i-1}}$ ($H_{1i}$ and $H_{2i}$ – lower and upper limit of the specified thickness range, respectively).

The ice survey data can also be used to plot bar charts of partial distributions for the ice cover linear size and concentration. In this study the total number of linear dimension ranges and their limits are chosen based on the regulated width of typical drifting ice formations: brash ice (less than 2 m across), ice cake (2 - 20 m), small floes (20 - 100 m), medium floes (100 - 500 m), big ice floes (500 - 2000 m), vast floes (2000 - 10000 m) and giant ice floe (over 10000 m). No correlations have been found between the horizontal size and thickness of ice cover, therefore generation of separate (independent) bar charts of partial distributions for these two parameters of ice conditions does not contradict the essential principles of simulation modeling.

Regarding the bar chart of partial distributions for ice concentration it is found that five grades are quite sufficient with the following descriptions: consolidated ice, very close pack ice 10/10 – 9/10 (floeS frozen together), close pack ice 8/10 -7/10 (floeS mostly in contact), open pack ice 6/10 -4/10 (floeS generally not in contact), very open pack ice 3/10 – 1/10 (water preponderates over ice). The method considered for estimation of ice loads on ship hull directly takes into account the ice cover concentration only in simulating the ship in ice operating mode and ship design speed in ice at the specified zone/route, i.e. in finding the qualitative relationship between the ship speed in ice versus ice thickness.

Random values (realizations) of linear dimension and concentration of ice cover can be obtained from given charts of partial distributions as in the case of ice cover thickness using the above given algorithm.

Fig. 1 to 3 give the examples of partial distributions charts for various ice condition parameters at the Northern Sea Route in the Barents Sea (Kolguev Island– Kara Gate Strait) and in the East Siberian Sea (72°00’ of north latitude/163°00’ of eastern longitude – 71°00’ of north latitude/170°00’ of eastern longitude).

Fig. 1. Chart of ice thickness partial distributions.
The total amount of snow on ice is commonly characterized by the geographical and seasonal factors and, as a rule, shall be determined by the snow cover thickness. These values are widely variable depending on where the snow cover melts the brine, which is heavier than ice, gradually flows down through cracks while the spaces between crystals are filled with air bubbles. For this reason the sea ice which has melted at least during one summer season features lower salinity than the young and first-year ice, while the multi-year ice is almost entirely fresh.

The salinity of first-year ice (thickness 0.30 to 1.80 m) is most commonly found using the following empirical formula (Ryvlin, 1974):

\[ S_i = S_{0} (1 - b) \cdot e^{-aH_i} + b \cdot S_{0}, \]

where \( S_{0} \) is the average sea water salinity frozen to form the ice cover, \( \% \); \( H_i \) is the ice cover thickness, m; \( b \) is the constant non-dimensional factor equal to the ratio of the ice salinity at the end of its winter season growth cycle and the average sea water salinity. The factor \( a \) accounts for the effect of ice growth rate on ice salinity and varies in the range from 3.0 (ice growth rate over 4.0 cm per day) to 6.0 (ice growth rate less than 0.5 cm per day). In this study the factor \( a \) is assumed to be uniformly distributed in the specified range and it is generated by standard generator of pseudo-numbers.

The specific thickness-averaged brine content in sea ice can be estimated with sufficient accuracy for practical purposes versus ice temperature \( T_i \) and ice salinity \( S_i \) using the following equation (Frankenstein and Garner, 1967):

\[ \nu_b = 0.001 \cdot S_i \left( \frac{49.185}{T_i} + 0.532 \right). \]

The sea ice cover density is primarily determined by its porosity and salinity, it should be noted that the porosity has a much greater influence on the ice cover density than the ice salinity. The ice cover density weakly grows with the brine content and steeply falls with the increase of air bubble content. A typical range of ice cover density variations is from 0.84 t/m\(^3\) to 0.94 t/m\(^3\), note that lower-end values correspond to summer-autumn season, while the higher-end values correspond to the winter-spring season (when the temperature of the ice cover decreases its density increases). The available analytical expressions can be used to estimate the ice cover density versus ice temperature, specific content of liquid and gaseous phases in ice, etc. In this study a simplified approach is used: the ice cover density is considered as a random value uniformly distributed over the specified range and it is modeled by standard generator of pseudo-numbers.

According to the experimental data the ultimate bending strength of ice \( \sigma_f \) varies in a wide range in function of ice cover temperature and salinity; however this value is practically not dependent on the load application time. The average values of \( \sigma_f \) obtained from static tests of floating ice cover in natural conditions are 0.50 to 0.70 MPa for strong winter-season saline ice and 0.20 to 0.30 MPa for summer-season saline ice weakened by melting (Ryvlin and Kheisin, 1980).
Among many experimental techniques proposed for determining the ultimate bending strength of ice the most reliable method today is breaking of ice-cut beams and cantilevers afloat. Unlike small-size specimens this method provides practically the same results across a range of different research studies with quite a moderate scatter of \( \sigma_f \) values. Also, the ice bending strength determined by tests on small-size specimens (1.00 to 2.00 MPa) is significantly higher than a similar value determined from tests on floating ice beams and cantilevers.

The empirical dependence between the ultimate bending strength of ice \( \sigma_f \) and specific content of brine \( \nu_b \) obtained after processing and generalization of experimental data is as follows (Doronin and Kheisin, 1975):

\[
\sigma_f = 0.70 \left( 1 - \frac{\nu_b}{0.202} \right) \pm \Delta \sigma_f. \tag{5}
\]

Allowance \( \Delta \sigma_f \) accounts for the actual scatter (i.e. deviation from formula-based values) of the parameter \( \sigma_f \) values obtained from tests. It is within 0.10 MPa. In this study the allowance \( \Delta \sigma_f \) is assumed to be uniformly distributed in the specified range (from \(-0.10\) MPa to \(+0.10\) MPa) and it is generated by standard generator of pseudo-numbers.

Fig. 4 shows the results of modeling the ultimate bending strength of ice for the Northern Sea Route in the Barents Sea and East Siberian Sea.

Ice compression strength \( \sigma_c \) is usually determined experimentally by crushing ice pieces of standard size and cubic or similar to cubic forms using special test rigs. It should be noted that \( \sigma_c \) is significantly dependent on the velocity and direction of load application to specimens, characteristic dimensions of specimens, their structure, temperature, salinity, crystal orientation with respect to the optical axes, etc. In accordance with the experimental data, the values of \( \sigma_c \) under static loading range from 0.5 MPa to 1.0 MPa (Popov, Faddeev, Kheisin and Yakovlev, 1967). However, the values of \( \sigma_c \) obtained by tests when the ship hull interacts with ice cover should be treated exclusively as certain measures of local ice crushing strength. These values cannot be used directly (without processing) to determine ice loads on ship hull because the ice failure pattern in the experiment is essentially different from the pattern of ice edge crushing caused by ship side penetration into ice. The main difference is that in case of tests on ice specimens a uniaxial compression is applied, while the ship hull/ice cover interaction involves constrained deformation (biaxial compression). It should be noted that the compression strength of upper ice layers, which are mainly locally crushed by an inclined side of ship, is quite low in the summer season due to melting under solar radiation, while in the winter season it proves to be much higher than that of ice specimens due to constrained deformation. As a rule, the constrained deformation effects are taken into account by introducing an appropriate factor of proportionality. According to approximate estimates of Korzhavin the constrained deformation increases the ultimate strength of ice obtained under uniaxial compression of ice specimens by a factor of 2.5 to 2.7 (Korzhavin, 1962).

Due to the effects of numerous and various factors the ultimate ice compression strength \( \sigma_c \) varies in a rather wide range (according to some sources up to 8.0 -10.0 MPa and even up to 25.0 MPa in case of triaxial compression) (Navwar, Nadreau, and Wang, 1983). In the engineering calculations it is assumed that among the multitude of factors influencing the value of \( \sigma_c \) it is sufficient to consider only the temperature and salinity of ice cover. It is also quite important to investigate how \( \sigma_c \) is influenced by the loading rate because under dynamic loading the ice compression strength is increased several times as compared to static values.

It is known that the ultimate crushing strength of ice \( \sigma_c \) is higher than the ultimate bending strength of ice \( \sigma_f \), while the strength of ice cover are mainly determined by ice temperature and salinity: the ice strength is increased as the temperature is reduced and the same is reduced as the salinity is increased. In this study a linear relationship between \( \sigma_c \) and \( \sigma_f \) is proposed:

\[
\sigma_c = k \cdot \sigma_f. \tag{6}
\]

Based on the estimates of various authors the range of proportionality factor \( k \) is limited to 1.5 - 6.0. This factor is considered as a random value uniformly distributed in the specified range of values and it is simulated by standard generator of pseudo-random numbers.

Fig. 5 shows the results of modeling the ultimate compression strength of ice for the Northern Sea Route in the Barents Sea and East Siberian Sea.
side structure strength under interaction with ice, which was performed for the existing ice-class merchant ships.

Fig. 6 shows the results of modeling the conditional parameter of dynamic ice crushing strength for the Northern Sea Route in the Barents Sea and East Siberian Sea.

Fig. 6. Conditional parameter of dynamic ice crushing strength: modeling results.

The elastic properties of ice cover represented as a plate are described by some averaged values of Young’s modulus $E_i$ and Poisson's constant $\mu_i$. The most common experimental techniques for finding the above-mentioned characteristics of ice cover are acoustic and seismic methods as well as tests of small-size ice specimens.

The value of $E_i$ significantly depends on the deformation time and temperature of ice cover and to a lesser degree on the ice salinity. According to the experimental data the Young’s modulus for strong saline Arctic ice under dynamic loading ranges from 2.0 GPa to 4.0 GPa (Popov, Faddeev, Kheisin and Yakovlev, 1967). These values of $E_i$ were obtained by measuring the resonance (critical) frequency of bending waves propagating in ice cover and these correspond to average values of Young’s modulus over large area. Under static loading $E_i$ significantly depends on the ice deformation rate, and it should be noted that at loading rates less than ~0.1 MPa/s one can clearly observe plastic behaviour of ice cover mainly caused by creep effects. The Young’s modulus of sea ice under long-term loading proves to be by an order of magnitude lower as compared to the dynamic modulus of elasticity.

The Poisson constant $\mu_i$ for sea ice is a relatively stable value ranging from 0.33 to 0.37 (Ionov and Gramuzov, 2001).

In this study the ice cover elasticity characteristics, viz. Young’s modulus and Poisson constant, are assumed to be random values uniformly distributed over respective ranges (2.0 - 4.0 GPa for $E_i$ under dynamic loading and 0.33 - 0.37 for $\mu_i$), and these are simulated by standard generator of pseudo-random numbers.

During the interaction of ship hull with ice the ship side impacts against the ice cover edge whose geometry in contact area can be qualitatively different. In the model used in this study to determine the ice loads on ship hull it is assumed that the ice edge has a standard (simplified) shape: either rounded or angular shapes. The crushed area due to cutting of a rounded or angular ice edge by the ship side plane is accordingly represented by a parabolic or triangular segment. In this study the edge shape choice is stochastic and made in the assumption that the probability of impact on a rounded edge $P_{\psi} = 1 - P_{R}$ are equal, i.e. these are equally likely events $P_{\psi} = P_{R}$. For modelling the ice cover edge geometry in the ship/ice impact area a series of random-number sequence is used with random numbers being uniformly distributed in the range (0, 1).

The rounded ice edge is described by radius $R$, while the angular edge is described by corner angle $\psi$. These parameters have little influence on the ice load magnitude, therefore these can be simulated approximately using basic approaches. According to the available statistics the acceptable curvature radius values range from 10 m to 40 m, while the corner angle observed in reality range from 45° to 145° (Popov, Faddeev, Kheisin and Yakovlev, 1967). In this study the parameters $R$ and $\psi$ are assumed to be uniformly distributed over the specified ranges and these are generated by standard generator of pseudo-numbers. Instead of the parameters $R$ and $\psi$ it is possible to simulate the values $(2 \cdot R)$ and $(tg \frac{\psi}{2})$ which are directly included in respective expressions for estimation of ice loads.

**Modeling of Ship Operation Mode in Ice and Designed Ship Speed in Ice**

In view of the significant number of various factors determining the ship motion in ice conditions its modeling is the most complicated and laborious task. The specifics of ship motion process in ice and the requirements laid to ship hull strength enable to mark out some typical ice navigation modes. This study considers the following modes for design ship:

- an independent motion in ice or a motion in a channel behind an icebreaker (the mode is set as initial information);
- a continuous motion in ice or a ramming tactics (the mode is installed depending on ship speed in ice);
- a motion in compact ice or a motion in broken ice with different concentration (the mode is installed depending on ice condition parameters).

When the ship speed in ice is modelled it is suggested to use simplified (linear, piecewise-linear etc) relationships linking this motion characteristic and ice cover thickness. The influence of ice cover hummocking, fracturing and snow-covering isn’t taken into account at the present stage.

When the ship is sailing in compact ice the ship speed is determined as a rule according to the following well-known formula:

$$\nu = \nu_0 \cdot \left(1 - \frac{H_i}{H_{lim}}\right),$$

where $\nu_0$ is the ship speed in open water, kn.; $H_i$ is the ice cover thickness, m; $H_{lim}$ is the ultimate thickness for level compact ice which is run by continuous ship motion, m.

When the ship is sailing in broken ice the ship speed can be estimated depending on ice concentration in accordance with the following numerical expressions:

$$\nu = \frac{\nu_{1k} - \nu_0}{H_{1k}} \cdot H_i + \nu_0, \text{ if } H_i < H_{1k};$$

$$\nu = \frac{\nu_{2k} - \nu_{1k}}{H_{2k} - H_{1k}} \cdot H_i + \frac{\nu_{1k} - \nu_2}{H_{2k} - H_{1k}} \cdot H_{1k}, \text{ if } H_{1k} \leq H_i < H_{2k};$$

$$\nu = \frac{\nu_{2k} - \nu_{1k}}{H_{limk} - H_{2k}} \cdot H_i + \frac{\nu_{2k} - \nu_{limk}}{H_{limk} - H_{2k}} \cdot H_{limk}, \text{ if } H_i > H_{2k},$$

where $H_{limk}$ is the ultimate thickness for level broken ice which is run by continuous ship motion, for $k$ -th rank of specified bar chart for ice concentration partial distributions, m; $H_{1k}$, $H_{2k}$ are the values of ice cover thickness which determine a location of intermediate range
bounds in accepted expressions for \( k \)-th rank of specified bar chart for ice concentration partial distributions, \( m \); \( v_{1k} \), \( v_{2k} \) are the values of ship speed in broken ice corresponding to the values of ice cover thickness \( H_{1k} \), \( H_{2k} \).

A piecewise-linear relationship between ship speed in broken ice and ice cover thickness was obtained by analysis, processing and approximation of full-scale trial data on a large-tonnage ice-going cargo ship by adherence of physical principles for real ship motion process in ice conditions.

The given expressions intended for determination of ship speed in ice specify only its certain average value. However, icebreaking process discreteness, ice cover non-uniformity, ship heave and pitch in ice as well as other factors cause stochastic speed fluctuations (Ryvlin and Kheisin, 1980). No information is available regarding the distribution function of speed fluctuations, but the great number of independent stochastic factors causing such fluctuations allows us to assume that these are distributed by the normal law characterized by following probability density:

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad (12)
\]

It is known, that practically all values (more than 99.7%) of normally distributed stochastic quantity are contained in the range \( \pm 3 \cdot \sigma \). An estimation of mean-square distance for ship speed in ice including fluctuations can be done supposing all its values are in sector restricted by straight lines \( v_1 = v_{00} \cdot (1-H_i/H_{lim1}) \) and \( v_2 = v_{00} \cdot (1-H_i/H_{lim2}) \):

\[
\sigma_v = \frac{1}{3} \frac{H_{lim1}}{H_i} \cdot \frac{v_{00} - v_{min}}{v_{min}}, \quad (13)
\]

where \( v_{min} \) is the minimum speed of steady ship motion in ice, kn.

When the ship is operating in ice with the speed \( \nu \geq \nu_{min} \) its motion remains steady and the stoppage probability is negligibly small. However, in modelling the stochastic values of ship speed in ice including fluctuations distributed by the normal law it is formally possible to obtain positive values lesser than \( \nu_{min} \) as well as negative values \( \nu < 0 \). These results can be interpreted as ship stoppage.

After ship stoppage the operator applies the ramming tactics, afterwards the continuous motion in ice is either restored or not. The ramming speed is determined by subjective factor in a considerable measure: the operator sets an upper speed limit reasonably in order to avoid extensive hull damages and ship jamming in ice during the impact interaction. An average value for ramming speed doesn’t exceed 4.0-6.0 kn. usually for ships with low ice category and can reach 10.0-11.0 kn. for ships with high ice category and icebreakers (Ionov and Gramuzov, 2001). Based on mentioned specifics of ramming tactics process it is expected that the ramming speeds are distributed according to the normal law.

**Modeling of Frequency Distribution for Impact Impulses over Ship Hull Length**

Full-scale ship trials and observations indicated that the location of ship hull/ice contact point is stochastic, while the occurrence of ice cover at any section between ship central longitudinal plane and its half-breadth is equiprobable. Thereby, if the initial ice state isn’t distorted by ship, the uniform distribution of ice cover over hull breadth would be considered true and the probability density of impacts’ number over hull length would be accepted proportional to \( \sin \alpha \) (\( \alpha \) is the angle of waterline inclination to ship central longitudinal plane).

In fact during the interaction between ship side and ice the distribution of impacts’ number over hull length is transforming and becoming more complicated. By strain-gage tests it has been established that the main factor governing the distribution of impacts’ number over hull length under rectilinear ship advance in ice is the waterline shape. Moreover, according to the experimental data the changes in ice conditions don’t exert a considerable influence on the type of impacts’ number distribution over hull length, while the maximum probability density of impacts’ number falls in the ship area of maximum waterline curvature:

\[
K = \frac{d^2 y}{dx^2} \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{-3/2}, \quad (14)
\]

where \( y = f(x) \) is the equation for waterline corresponding to design ship draft.

When the distribution of impact impulses’ frequency is modelled it is suggested in this study to use the following technique. To obtain quite close to actual distribution of impacts’ number over hull length it is advised to introduce a certain fictitious distribution of contact points over hull breadth that would cause a distribution of impacts’ number over hull length being completely similar to the experimentally determined distribution. If the specified fictitious distribution is replaced by trapezium whose ordinate \( b \) is chosen herewith in way that the coordinate of maximum probability density of impacts’ number coincides over hull length with the coordinate of maximum waterline curvature, then the modeling of non-dimensional contact point coordinate over hull breadth can be performed using standard pseudorandom number generator:

\[
y(\xi) = -\frac{1}{2} b \frac{1}{1-b} + \sqrt{\frac{1}{4} \left( \frac{b^2}{1-b} \right)^2 + \frac{\xi}{1-b}}, \quad (15)
\]

where \( y = y(B/2) \) is the non-dimensional linear coordinate finding the position of ship hull/ice contact point concerning central longitudinal plane and corresponding to design waterline half-breadth; \( \xi \) is the random number distributed uniformly in the interval \((0, 1)\); \( b \) is the non-dimensional parameter; \( B \) is the design ship hull breadth, m.

To proceed to the sought-for distribution of impacts’ number over hull length it is necessary to calculate the stochastic values of dimensional coordinate \( x \) finding the contact point position lengthwise per stochastic values of non-dimensional coordinate \( y \) by means of given equation for design waterline \( y = f(x) \) and then to group the obtained values relative to the specified sections over hull length.

**SIMULATION RESULTS FOR ICE LOADS ON SHIP HULL**

In this study the simulation of ice loads is performed in respect to the hull of Samotlor-type ice-going oil tanker. The specified ship of about 17200 tons’ deadweight was designed under a class of Russian Maritime Register of Shipping and has UL ice category. It is intended both for independent operation in ice and for operation in a channel behind an icebreaker on the lines of Northern Sea Route in west and east parts of Russian Arctic. It is considered in test examples that the ship moves ahead and rectilinear in a channel behind an icebreaker on the following two lines of Northern Sea Route: in the Barents Sea (Kolguev Island – Kara Gate Strait) and in the East Siberian Sea (72°00’ of northern latitude/163°00’ of eastern longitude – 71°00’ of northern latitude/170°00’ of eastern longitude). The seasonal navigation period is April.

The ice loads’ parameters are determined with the use of known dependences of hydrodynamic solid body/ice impact model. The accepted impact model is based on the solution of task about an
extrusion (squeezing) of the relatively thin intermediate layer from power contact zone to free surface during the ship side penetration into ice. This layer is notable for small-dispersed structure and has both viscous and plastic properties at the same time. The following ice loads’ parameters are modeled in test examples:

– ice load intensity which characterizes a maximal pressure value in power contact zone between ship hull and ice;
– height of ice load distribution which characterizes a maximal transverse dimension of power contact zone between ship hull and ice;
– linear ice load.

The random values (realizations) of ice loads’ parameters are obtained during the probabilistic computer experiment. The special-purpose software developed by the authors is used directly for its implementation (Dudal, Yakimov and Tryaskin, 2010). The initial distributions for specified stochastic parameters are ascertained experimentally and presented in the bar charts’ form. Afterwards these distributions are approximated by continuous two-parameter gamma distribution having the following view of probability density for nonnegative random values:

\[ f(x) = \frac{1}{\theta^k \Gamma(k)} x^{k-1} e^{-x/\theta}, \quad x \geq 0 \]

where \( \Gamma(k) = \int_0^\infty t^{k-1} e^{-t} dt \) is the Euler gamma function; \( \theta > 0 \) is the scale coefficient; \( k > 0 \) is the shape coefficient.

Fig. 7 to 12 give the modeling results for ice loads’ parameters.
CONCLUSION

The paper presents the results of research the authors, conducted over the past 4 years. The proposed method of stochastic simulation of ice loads has shown its effectiveness. The considered simulation model is implemented for the relative simple problem - determination of ice loads on the ship hull. Much more complicated is the inverse problem - the construction of the laws of distribution of admissible velocities of ships in ice, solution of which the authors suggest to be completed in the near future. Proposed physical model of the ship hull with ice interaction gives possibility to solve this problem. At the same time it is possible to take into account probability characteristics of strength of the hull structures due to corrosion and mechanical wear, variability of strength characteristics of steel, deviations from the actual values as-built (design) dimensions of structural members. The solution of this problem will allow reasonably approach to standardization of the ice strength of vessels for ice navigation including advanced large-tonnage vessels, to development of certificates on the safety of ships navigating in ice - special documents governing the permissible conditions of operation of the vessel in ice, which are implemented by Russian Maritime Register of Shipping in the practice (Tryaskin, Didkovskiy, Kuteinikov, Grubov, Andryushin 2009, Tryaskin, Yakimov and Kuteinikov, 2010).

REFERENCES