Multi-variate I-FORM contours for the design of offshore structures
(Practical methodology and application to a West Africa FPSO)

Michel François, Claude Camps, Juan Alvarez,
Bureau Veritas - Marine Division
Paris, France

Valérie Quiniou,
TOTAL S.A.
Paris, France

ABSTRACT

With the refined multivariate description of metocean conditions now becoming available, "Response-Based Design" methodologies can be developed, to address the multivariate nature of the response of floating offshore structures. In this paper, the Inverse First Order Reliability Method (I-FORM), derived from structural reliability techniques, is considered to build a practical methodology that can be implemented using standard engineering tools.

The steps and methods to build the contour for a multivariate environment are reviewed. The approach of "polygonal contours" is developed, that practically appears as an extension, and a rationalisation in a Response Based context, of the traditional concept of "associated value(s)". A strategy for scanning through polygonal contours is presented. Examples of contours and their application to a spread-moored FPSO in West Africa are presented for illustration of the method.

KEY WORDS:
FPSO; Metocean; I-FORM; West-Africa, station-keeping;

NOMENCLATURE

X, Y, ... metocean parameters (physical variables)
F(X) cdf (cumulative distribution function) of X
H(X, Y, ...) system response
G(X,Y,..) limit state function
P(A) probability of the event A
p(X) pdf (probability density function) of the variable X
pe(X) probability of exceedance of X
R(X, Y,...) Rosenblatt transform
[ u , v , ...] the set of independent standard Gaussian variables representing [ X , Y , ...]
β target safety index β
θ, dir direction (continuous variable or sector)

INTRODUCTION

Marine Structures are subject to the actions induced by marine environment (waves, wind, current, and related effects). Here, the extreme (low probability) response is addressed, from the view point of the metocean conditions that may induce such response.

The traditional method to evaluate the extreme response of a structure has been to calculate the response for the extreme value (i.e. the value with a specified probability of exceedance / Return Period) of a governing metocean intensity parameter (e.g. wave height). This methodology is appropriate when the response is governed by a single parameter (e.g. wave height for jackets), but indeed only applicable under such condition.

Considerations of the multivariate nature of the response (particularly for floating structures) have motivated the development of "Response-Based Design" (RBD) methodologies, i.e. aiming at a specified probability of exceedance / Return Period of the response (see e.g. Haver 04).

In the meantime, refined descriptions of weather conditions are now becoming available, generally showing (e.g. in West Africa) more complex conditions than pure wind driven seas, also motivating further developments of RBD methodologies towards practical engineering tools.

One practical approach is the I-FORM "Contour" approach, that has been introduced (Winterstein 1993) and principally used until now for the [Hs, Tp] response, or for only a small number of variables (see also Haver 2004).

As a continuation of an effort to incorporate response-based directional criteria in the assessment of floating structures and their station keeping system (see François 2004(a) and BV 2004), the work presented in this paper was performed to draw the path for implementing this I-FORM method in the design or assessment process of floating structures (the floater it-self, its station-keeping system, risers, etc...), in the context of a multivariate description of metocean conditions. This work is part of a study of “Joint Probabilities of Wind/ Wave/ Current and Response-Based Design of FPSO, Moorings and Risers” funded by TOTAL S.A.,
R&D division, with the support of TOTAL E&P Angola. Focus was on practical implementation issues and, based on the extensive analysis of Angola Block 17 data performed within the present project (Nerzic 2007), on the metocean conditions prevailing in West Africa waters.

The steps and methods to build contours for a multivariate environment were reviewed in detail. Some outstanding issues are outlined in this paper, starting from the distributions of parameters until the elaboration and use of the multivariate contour. An approach of "polygonal contours" is developed and a strategy for scanning through polygonal contours is presented. Specific attention is given to the modelling of directional data. Examples of contours and their application to a spread-moored FPSO in West Africa are presented for illustration of the method.

FORMAT OF DATA AND RESPONSE ASSESSMENT

Short-term and Long-term Conditions and Response

Short-term Conditions: The weather conditions at a given site and time are characterised by a set of metocean parameters (wave height and period, wind and current speeds, directions, ...) that are generally averages (or other statistical quantities) over a short duration (typically 10' to 3h), associated to a model of the short term variations (e.g. wave spectra for water surface).

With an analysis of the system response, the short-term response is obtained, taking into account these models, and generally assuming that the process is stationary.

Long-term Conditions: The slowly varying nature of weather is described by the time series of metocean parameters representing each successive 'metocean event' (e.g. sea-states) encountered by the structure, or by the statistics of these parameters. The total number of 'metocean events', needed to define a probability, is determined by the duration of (or the time interval between) records, e.g.:

\[ N_{sy} = 2922 \text{ per year, for 3 h events} \]

Response: A response event can be a statistical value (e.g. mean or most probable maximum) over the duration of the 'metocean event', or the (random) maximum over same period (same number \( N_{sy} \) as the metocean event, in both cases). In this paper, the probabilities are taken, unless otherwise noted, as the probability per sea-state, and are generally expressed in this report as a probability of exceedance:

\[ p_e (X) = p(x > X) = 1 - F(X) = 1 / (N_{sy} \cdot RP) \]

where \( RP \) is the return period, i.e. the reciprocal of the "annual probability" (of exceedance) referred to in some standards (such as ISO 19901-1).

Note: When \( X \) is a (short term) maximum response, \( p_e(X) \) should in principle include the short term variability of the response (see Haver 2004). This aspect is however not covered in the present work.

FORM and I-FORM

FORM/SORM: The FORM (First Order) and SORM (second Order) Reliability Methods may be used to assess the dependence of a structural response on the set of random variables defining 'metocean events'. A very brief outline is given below. Formal descriptions are available in textbooks (see e.g. Lemaire 2005).

With an analysis of the system response, the short-term response is spectra for water surface).

\[ 10' \text{ to } 3h \text{), associated to a model of the short term variations (e.g. wave averages (or other statistical quantities) over a short duration (typically 10' to 3h), associated to a model of the short term variations (e.g. wave} \]

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\[ \text{[X, Y, ...] being the set of metocean parameters (physical variables), defined by their joint distribution, and H (X, Y, ...) being a (scalar) function defining the system response, a “limit state function” G is defined by:} \]

\[ G(X,Y,...) = R - H(X,Y,...) \]

\[ \text{where R is the response level of interest.} \]

\[ \text{Using the transform} g \text{ of G in a space of independent standard Gaussian variables, } [u, v, ...] \text{ representing } [X, Y, ...] \text{(Rosenblatt transform), FORM provides the (first order) safety index } \beta \text{, thus a first order estimate of the probability of exceedance of the response (as the "probability of failure" } pf \text{ of the system):} \]

\[ pe(R) = pf \approx pf1 = \Phi(\beta) \]

\[ \text{where } \Phi \text{ is the cdf (cumulative distribution function) of a standard Gaussian variables.} \]

\[ \text{A second order approximation } pf2 \text{ is given by SORM, that is taking into account the curvature of } g \text{ (the transform of G in the u,v,... space). A more accurate } pf3 \text{ can be obtained by Monte Carlo simulation with appropriate sampling technique (e.g. the "importance sampling"). There are however some difficulties in case of multiple work points, a situation that might not be avoidable and will need to be carefully reviewed. FORM also provides the “work point” (or “design point”), that is the most pertinent combination of the variables X, Y, ... for which G(X,Y,...) = 0.} \]

\[ \text{It is worth noting that FORM, as well as SORM or I-FORM are not using the above transform itself, but the inverse transform} \]

\[ (X,Y,...) = \mathcal{R}^{-1}(u,v,...) \]

I-FORM: With FORM/SORM, the \( \beta \) for a given H and a value of R is evaluated. With I-FORM, a contour (hyper-surface) is defined, from which the response corresponding to a specified target \( \beta \) can be evaluated as the maximum of the response H(X,Y,...) over the contour, thus turning the reliability problem into a standard scanning problem. The contour surface is, in the space of independent standard Gaussian variables, a (hyper-)sphere of radius \( \beta \). The contour in the space of the physical variable is obtained, as the inverse transform of this (hyper-)sphere, and is independent of the response function H:

\[ \text{Thus no prior knowledge of H is needed to build the contour. This is however a first order approximation. For a given H, calculation with FORM/SORM will give } \beta1 (= \beta) \text{ and } \beta2 \text{ (or } \beta3 \text{), from which the contour could be adjusted (but the correction could then depends on H).} \]

DISTRIBUTIONS OF METOCEAN PARAMETERS

Type of Parameters

Intensity parameters: Intensity parameters (e.g. the (significant) height of waves, the wind or current velocity, ...) are characterised, in the first place, by a cumulative marginal distribution F(X). A practical representation is X as a function of the probability of exceedance pe(X) with a log-scale for pe, as typically used by engineers to draw long-term distributions of the response.

Other parameters: For other parameters (e.g. direction, period, ...) a RP have no sense, but using a cumulative distribution (without log scale) can be convenient for analysis or for presentation.

Correlations and joint-probability distributions

Correlations: Once a set of parameters has been identified to represent the (short term) metocean conditions at a given site, assessing correlations between the corresponding variables is an important step in
the building of joint probability distributions.

The degree of correlation may vary from none (e.g. swell and current in West African climate) to high (e.g. wind and wind sea in North West Europe climate), depending on elements and location. Among the methods to evaluate correlation, linear correlation, or Plackett correlation (for 2 variables), or Principal Component Analysis (PCA) for several variables, will give a first information on the degree of correlation between variables, but will not provide further element for modelling, as the physics behind the correlations, e.g. wind sea and wind, is usually non-linear.

The conditional means can be used to demonstrate independence (in theory, as some scatter is always present in the tails of distributions), or evidence correlations, then provide initial data for modelling by conditional distributions (see Fig. 1 below).

For several variables, the matrix of pƎ should match the independence (or dependence) assumptions.

On the other hand, when an element is always present, with no calm (e.g. swell in West Africa), the (fitted) distribution should match Xmin corresponding to pe(Xmin) = 1.

Other conditions may arise from e.g. the partitioning of complex sea-states in distinct components, that should be duly identified, but it may be currently difficult to formulate such conditions in a better way than a rather crude modification (truncation) of contours. This area would require further investigations, in relation with the development of the partitioning techniques themselves.

Extrapolation to extremes

In I-FORM, the contour is extracted from the (joint) distributions of parameters including all metocean events. Therefore the extrapolated intensity parameters shall represent an extrapolation of the (marginal) distribution of parameters. However, other techniques are generally used to derive extreme values. Then, for the consistency of contours with these data, the fitted distribution should match these values. There are however theoretical difficulties that would require further work.

The joint distributions of non-independent parameters also have to be extrapolated so as to represent a joint distribution of all (low probability) events. This should be made based on the observations from the metocean data base or other appropriate considerations, with Oceanographer’s judgment, combined with Engineer’s knowledge of the impact on response.

Often argued as a weakness of the method, this is indeed its strength: Avoiding any a priori assumption on the multivariate nature of the response, or the sensitivity to any “secondary” parameter, the computation of the response along the contour will highlight this nature, whenever significant, and quantify it. Then, there could be case for a closer look to the data-base, in the light of a particular response.

2D & 3D CONTOURS

Some aspects of the "classical" bi-variate contours and considerations on 3D contours are outlined below, as a step towards multivariate contours, and before considerations of simplifications with polygonal contours.

2D contours

For two variables the bi-variate contours are, for a set of return period the transforms of concentric circles in the [u, v] space. Each contour can be labelled with the RP (or pe) corresponding to the β of the same contour in the [u, v] space (see examples in Nerzic 2007).

The shape of contours will depend on the marginal distributions of the variables and, critically, on the correlation between variables, hence the above discussion.

For an intensity variable X, and whatever the correlation, the maximum of X along the contour with the “label” RP is the value XRP with the return period RP, following the marginal distribution of X. This point is the work-point when the response is function of the variable X only (e.g. H(X,Y) = X).

The value of Y at this point is the 50% quantile (the median) of the marginal distribution of Y when Y is independent of X, or of Y | X when Y is depending on X (but a different value if the dependence is described by X | Y).
When a variable \( Y \) is present only a part of the time, the \([X, Y]\) contours will be modified. In particular, the above point at \( X = X_{rp} \) will be at \( Y = 0 \), if \( p \) is 50% or lower.

**3D contours**

Similarly to bi-variate contours, the contours (surfaces) for 3 variables and a set return periods are the transforms of concentric spheres in the \([u, v, w]\) space, having similar characteristics as noted before for 2D contours.

**Dependent variables \( X, Y|X, Z|Y \):** In the case of a 2 level dependence, such as \( Y \) depending on \( X \) and \( Z \) depending on \( Y \) (e.g. in the case of wind, wind sea height, and wind sea period), the surface forming a 3D contour can be described by a set of 2D contours \( f(Y,Z) \) for a given \( X \), as illustrated in fig. 2 below for the case of \( V \) (wind velocity), \( W \) (wind sea Hs) and \( T \) (Tp of wind sea).

![Fig. 2: \([V, W, T]\) contour RP100 with iso V](image)

A 2D contour \( YZ \) based on the marginal distribution of \( Y \) may be also drawn, that is indeed the envelope of the projection of the surface on the \( YZ \) plane, i.e. combinations of \( Y \) and \( Z \) when the response is not depending on \( X \). Otherwise, the contours \( f(Y,Z) \) for a given \( X \) will generally have a different shape (see \( W \) & \( T \), in the figure above).

**Presentation of contours (the \( r \)-space)**

When a variable \( X \) is an intensity or a response, a value of \( X \) can quoted by its return period \( RP \), or the corresponding \( pe \), or the corresponding \( u = -\Phi( pe ) \):

\[
RP(X) = 1 / [ N . pe(X) ] \\
pe(X) = 1 / [ N . RP(X) ] = \Phi(-u_x)
\]

It is also convenient to use a log scale, i.e. \( r_x = \log_{10}(RP(X)) \). As \( r_x \) is, similarly to \( u_x \), a presentation of \( pe(X) \), it does not depend on the distribution of \( X \) (see table 1).

For several intensities, it is then convenient to represent the contour in the \( r \)-space:

For independent variables and a given \( RP \), there is a unique contour \([ r_x, r_y ]\) (or \([ r_x, r_y, r_z ]\) for 3 variables).

For dependent variables, the shape of the contour is depending on correlations and highlights the effect of these correlations, as illustrated in fig. 3 below.

**Table 1: correspondence between, \( RP, r, pe \) and \( u \) (for 3h events).**

<table>
<thead>
<tr>
<th>( RP ) (years)</th>
<th>( r = \log(RP) )</th>
<th>( pe )</th>
<th>( u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3.46</td>
<td>1</td>
<td>-\infty</td>
</tr>
<tr>
<td>0.0007</td>
<td>-3.16</td>
<td>50%</td>
<td>0.00</td>
</tr>
<tr>
<td>0.0068</td>
<td>-2.16</td>
<td>5%</td>
<td>1.64</td>
</tr>
<tr>
<td>0.01</td>
<td>-2.00</td>
<td>0.034</td>
<td>1.82</td>
</tr>
<tr>
<td>0.1</td>
<td>-1.00</td>
<td>3.4E-03</td>
<td>2.70</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>3.4E-04</td>
<td>3.40</td>
</tr>
<tr>
<td>10</td>
<td>1.00</td>
<td>3.4E-05</td>
<td>3.98</td>
</tr>
<tr>
<td>100</td>
<td>2.00</td>
<td>3.4E-06</td>
<td>4.50</td>
</tr>
<tr>
<td>1000</td>
<td>3.00</td>
<td>3.4E-07</td>
<td>4.97</td>
</tr>
<tr>
<td>10000</td>
<td>4.00</td>
<td>3.4E-08</td>
<td>5.40</td>
</tr>
</tbody>
</table>

**Fig. 3: Contours of 2 intensity variables in \( r \)-space**

a) uncorrelated \( r = -3 \) to +2  
b) effect of correlation \( r=2 \); 
with (blue - West Africa wind and wind sea) and without (yellow)

**Fig. 4: Contour of 3 independent variables in \( r \)-space**

**Scanning for response**

The response \( H \) can be usually assumed to be monotonic with an intensity variable \( X \). Then scanning over only one (the upper) half of the contour is normally needed. For two (or more) intensity variables, having generally a cumulative effect, only the fraction of contour \([ u>0, v>0 ]\) need be considered in general.

For other variables (e.g. period, or direction) the response \( H \) is usually not monotonic with the variable, and a full scanning over the possible range of variation is needed in principle.
DIRECTION

By the physics of metocean climate, the incoming weather has prevailing directions (most frequent direction, and directions of higher intensity, often distinct). Besides, the response of most structures is, in the first place, and before any other parameters, sensitive to direction. A proper modelling of the directions is therefore essential.

There are however several difficulties:
- to get joint distributions,
- to model the direction in an I-FORM context,
- to model intensities that are correlated (e.g. wind sea and wind).

In this case, it might be more appropriate to consider, for one of the parameters, a relative direction.

Joint distribution of intensity and direction

The traditional method to present the joint direction/intensity distribution has been to use directional distributions, i.e. the distribution of intensity by sectors, i.e. classes of directions.

This approach has some drawbacks, particularly for its utilisation in the FORM and the I-FORM methodology:
- the size of data base per sector is reduced (with possibly very few data in some sectors) : this may severely affect the accuracy of (and the confidence in) extrapolated extremes - with inconsistent results in some cases - and/or lead to no information being available for some directions,
- the extrapolated distributions may not sum-up to the marginal distribution of intensity,
- a "rescaling" of extremes is needed.

Another approach is to consider the conditional distribution of direction versus intensity (or intensity exceedance), i.e. write:

\[ p( X, \theta ) = p(X) * p(\theta | X) \]

or better, a semi-cumulative distribution, as described therein, and illustrated in figure 5 below:

\[ pe(X, \theta ) = pe(X) * p(\theta | (x>X)) \]

Fig. 5: Directional distribution (surface), with iso-H and iso-pe

It is unlikely that the distribution(s) p(\theta) or p(\theta | (x>X)) could follow an usual statistical distribution.

The following techniques of adjustment may be used instead:
- for a limited directional spreading : fitting of the reduced variable u(\theta) as a function of \theta, as e.g. in fig. 6 below (main swell in West Africa conditions; fitting by a 2nd order polynomial),
- for a directional distribution extending over 360° (i.e. \theta is a circular variable), writing:

\[ F(\theta) = (\theta - \theta_0)/360 + Z(\theta) \]

where \theta_0 is an arbitrary start point, and Z(\theta) a periodical function.

A fitting taking the periodicity of Z into account can be made, then the pdf can be obtained by derivation.

Modelling Direction in a contour approach

When a directional distribution covers a limited sector, and is unimodal, modelling \theta as an I-FORM variable can be considered, but might be inaccurate. For a directional distribution extending over 360°, then direction being a circular variable, the direction for cut-off (or for u=0) is arbitrary, and a modelling as a u is not possible.

Pending further development in this area, it is proposed to get the directional data of intensities from the above surface of the semi-cumulative distribution, following the technique of iso-pe, presented in François 2004(a). This technique is indeed a rationalisation of the traditional concept of directional extremes.

The iso-pe line is giving, for a given RP and each direction 'dir', the intensity \(X_{dir}(RP)\), with \(X_{dir}(RP) = X_{allDir}(RP)\) in the most probable direction of \(X_{allDir}(RP)\).

Then, for evaluating the response to a variable X, and the combination with other variables:
- the response for the return period RP is taken as the maximum of the response along the iso-pe line, i.e. the iso-pe line is treated as a contour,
- for the combination with other (independent) variables (see below), the FORM variable u corresponding to RP is assigned to \(X_{dir}(RP)\),
- for the combination with a dependent variable Y|X, (e.g. wave period depending on wave height, and possibly on direction), the 2D contour of the variables X and Y passing by \(X_{dir}(RP)\) is used.
MULTIVARIATE CONTOURS

Approach

With I-FORM, the "contour", i.e. the combinations of the metocean variables to be explored to get the response with a return period RP, being the maximum of the response over the contour, is the inverse transform of a (hyper)sphere, in the space of standard variables, with a radius \( \beta_p \) given by:

\[
\beta_p = \Phi^{-1}[1 / (N \times RP)]
\]

This writes:

\[
[X, Y, Z, ...] = \mathbb{R}^{-1}(u, v, w, ...)
\]

with

\[
u^2 + v^2 + w^2 + ... = \beta_p^2
\]

Both physical and corresponding standard variables may be generally split in independent subsets, such as:

\[
[X, Y, Z, ...] = \mathbb{R}^{-1}(u, v), \quad [Z, ...] = \mathbb{R}^{-1}(w, ...)
\]

To these subsets may be attached \( p, q, ... \) given by:

\[
p^2 = u^2 + v^2 \quad q^2 = w^2 + ... \quad ...
\]

and the quadratic sum may be then written as:

\[
p^2 + q^2 + ... = \beta_p^2
\]

Thus \( p, q, ... \) may be considered as independent standard variable each globally representing subsets of all the variables, e.g.:

\[
P = [X, Y] \quad Q = [Z, ...]
\]

As

\[
p = \beta_p, \quad \text{when} \quad q, ... = 0
\]

\( p \) has the meaning of the safety index of a response \( H \) that would depend only on the variables forming the subset \( P \).

The RP contour of variables \([X, Y]\) is thus a subset of the RP contour of all variables. Besides, any point inside the RP contour of variables \([X, Y]\) is also a point of the global RP contour.

The return period \( RP(X, Y) \) of the contour on which a point \([X, Y]\) is lying gives the contribution of \( X \) and \( Y \), from which the total contribution of other variables is obtained by:

\[
q^2 = \beta_p^2 - \beta(X, Y)^2
\]

This separation provides an approach for the scanning of variables, based on:

- the unique contour of independent variables,
- the specific 2D (or 3D) contours of correlated variables.

Several levels of subset may be used, so that the response might be evaluated, where possible, for each subset, and then combined with the response to another subset, until the final response is obtained.

A suggested tree of variables is shown in table below, for the case of West Africa conditions, based on the independence or correlation identified in another part of the study (see Nerzic 2007).

To each physical variable (a metocean parameter or a response) is corresponding a standard Gaussian variable. However the direction, when modelled as proposed above, is not a standard Gaussian variable but appears only as a parameter in the distributions of related variables.

Scanning

Exploring all combinations of the base variables for a given \( \beta_p \) would require a very large number of combinations having to be analysed. One option is to use a FORM type of algorithm to search for the design point, the response being available in an analytical form, as a Response Surface, as in Orsero 2007.

Table 2: Combination tree of metocean variables (West Africa)

<table>
<thead>
<tr>
<th>M</th>
<th>C</th>
<th>Dc</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass of water</td>
<td>current direction</td>
<td>Current direction</td>
</tr>
<tr>
<td>A</td>
<td>Vd</td>
<td>Dv</td>
</tr>
<tr>
<td>atmosphere</td>
<td>wind</td>
<td>wind direction</td>
</tr>
<tr>
<td></td>
<td>Dw</td>
<td>wind Velocity</td>
</tr>
<tr>
<td>R</td>
<td>Wind sea</td>
<td>Dv</td>
</tr>
<tr>
<td>Response</td>
<td></td>
<td>wind sea direction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>wind sea Ts*</td>
</tr>
<tr>
<td>D</td>
<td>Sp</td>
<td>Dsp</td>
</tr>
<tr>
<td>distant storms</td>
<td>main Swell</td>
<td>direction of Sp</td>
</tr>
<tr>
<td>S2</td>
<td>Sd2</td>
<td>direction of S2</td>
</tr>
<tr>
<td>Second. Swell</td>
<td>Hs2</td>
<td>Hs of S2</td>
</tr>
<tr>
<td></td>
<td>Ts2</td>
<td>Tp of S2*</td>
</tr>
</tbody>
</table>

* : non-independent variables
(dependence on direction omitted for clarity)

POLYGONAL CONTOURS

Approach

The approach of "polygonal contours", proposed in François 2004(a), consists in a simplification of the true contour, in which a polygonal contour is defined, forming an envelope of the "true" contour, so that the scanning could be limited to the summits of the polygon.

Practically, it appears as an extension, and a rationalisation in a Response Base context, of the traditional concept of "associated value" (but a different concept than the simultaneous value in some metocean specification).

With respect to a continuous contour, a discrete contour will tend to overestimate, or may underestimate the response. To determine the values that are proposed below, systematic checking were performed for typical response functions such as linear and quadratic combinations of intensity variables and a power law for period. FORM and SORM were used, with Comrel® software, to verify the achieved Safety Index. The second order \( \beta_2 \) was always found higher than the target \( \beta_p \), except for some very narrow ranges of parameters defining the response, not significant with respect to the variation of response with e.g. vessel draft. The polygonal contour thus compensate for the underestimate of response by the First Order contour. The error on response with respect to the First Order contour was found to be within a small interval [-0 , +10% ] in most cases; the true error is thus lower. The contours were also evaluated with the response of a typical FPSO mooring system (see below).

For directions, a polygonal envelope of the iso-pe contour can be defined using appropriate direction steps, preferably to the traditional broad ‘sector’ data, not dispensing in any case from a more accurate scanning if the steps are larger than say 15°.
Contours of uncorrelated variables in the r-space

As noted before, the contour of uncorrelated variables (be-it physical variables or “responses”) in RP axis does not depend on the distributions, and can be described by the values of variables rX, rY, rZ, … associated to variables X, Y, Z, ….

Polygonal contours are proposed, starting with a 2D contour, then a 3D contour build from the 2D contour, as shown in the figures below, that can be generalised in n.D contours.

"X only": Scanning through all such combinations will generally not provide the maximum response (at the return period RP), but may permit a first identification of the relative order of magnitude of the actions induced by the different elements, or the critical directions, as discussed later.

"X governed": the ‘X governed’ conditions, with $r = r_{ass}$ for all other (independent) variables, are the traditional combinations.

"X Y driven": for $r_{rp} = 2$, $r_{mid}$ may be taken as -0.15 , or conservatively 0 (i.e. a 1 year RP) , as shown on the figure 7 above (red point).

**Fig. 7**: Exact (yellow) and polygonal 2D contour of independent variables, in r-space (blue and red), for $r_{rp} = 2$ (RP 100 years)

These contours involve the combinations of the four following values of each r and the corresponding variable V (i.e. X or Y or Z, or …):

- $r = r_{rp}$, $V = V_{RP}$ (i.e. extreme V)
- $r = r_{1}$, with $r_1$ defined by $pe(V)=1$ ; $r_1 = -3.46$
  i.e. $V=0$ (no V) (or $V_{min}$)
- $r = r_{ass}$, $V = V_{ass}$ (the associated V)
- $r = r_{mid}$, $V = V_{mid}$ (as defined below)

from which the following type of combinations can be defined:

**"X only"**: $r_X = r_{rp} , r_Y = r_Z = ... = r_1$

i.e. $X=X_{RP} , Y=Z=...=0$

**"X governed"**: $r_X = r_{rp} , r_Y , r_Z , ... = r_{ass}$ (or $r_1$)

**"X Y driven"**: $r_X = r_Y = r_{mid} ; r_Y , r_Z , ... = r_{ass}$ (or $r_1$)

i.e. a combination with an equal contribution of X and Y (to the reliability index $\beta_{rp}$ )

**Fig. 8**: Polygonal contour of 3 independent variables in r-space

For a further refinement of the contour, other variables could be taken following a contour with $r = r_{ass}$ , that could be built similarly to the 2D contour in figure 7, omitting the middle point.

The points of the polygonal contour nearest to $r_X = r_{rp}$ will be then given by :

$r_X = r_{rp} , r_Y = r_{ass} = r_{ass}(2), r_Z = r_{ass}(3), ...$

when, for each point, the variables X, Y, Z, T, … are sorted in decreasing order of contribution,

The $r_{ass}(n)$ will form a suite that is quickly converging to $r = -3.16$ , i.e. the median value (pe(X) = 50%) of the variable.

In such case, the variable will have almost no contribution to the response. Then, $r = r_{ass}$ could be taken conservatively, as noted before, without having a noticeable impact on the calculated response.

This also indicates that using a median value for a “secondary” variable indeed assume that its effect is negligible. This is however a convenient assumption to limit the number of variables having to be scanned simultaneously, subject to a sensitivity analysis, or/and a later assessment.

**Scanning**: The total number of points on the polygonal contour of n independent variables is :

$$N_n = n*(n+3)*2^{n-3}$$ (e.g. $N_n = 18$ for $n = 3$ , $N_n = 160$ for $n = 5$)

However, with an organised scanning, the number of points may be limited to n for the “X only” cases , then at most $n*(n+1)/2$ for the “X governed” and the “XY driven” conditions ( i.e. 3+6 for $n = 3$ , and 5+15 for $n = 5$). More detailed assessments may be then limited to the regions found of interest, together with the assessment of other associated variables.

**Contours of correlated variables**

The contour simplification is depending on correlations. Two practical cases are presented below. As these contours are site specific, they are given only as illustration of the method.

**Contour of wave height and period**: The 2D polygonal contours of wave (significant) height $Hs$ and period $Tp$ , and three values of r, are shown below, for the case of West Africa main swell.
Further studies however highlighted that the conditional distribution of period is also depending on direction, and that a 3D contour $H_s - T_p$ direction was needed to catch the maximum response.

3D contour of wind velocity, wind sea height and period $T_p$ : The 2D contour of wind velocity $V$ and wind sea (significant) height $W$, shown before can be simplified as shown in figure 10a), with three points of practical interest:

a) max $V$ and associated (higher) $W$

b) max $W$ and associated (higher) $V$

c) max $W$ and associated lower $V$  
For $r = r_{ass} = -2.5$, a single point is defined as $r_V = r_W = r_{ass}$, i.e. as if $V$ and $W$ were fully correlated. The same simplification $r_V = r_W = r_p$ or $r_{mid}$ could be also used for a first step of scanning. The 3D contours, i.e. including wind sea period, and corresponding to the above three points, are shown for $r = 2$, in figure 10b.

Scanning

A strategy for scanning through polygonal contours could be summarised as follows, on the basis of the variables as identified in the tree of variables presented in table 2 (West Africa conditions):

Step 1 : “$X$ only" conditions, with $X = M$ or $A$ or $D$, taking into account the following:
For $X = A$ (wind / wind sea), $r_V = r_W = r_{ass}$ will be taken for $V$ and $W$, and the directions of $V$ and $W$, that are weakly correlated, should be assumed as independent.
For $X = D$ (swell), $D$ may be taken as the envelope, over directions, of main swell $S_p$ and secondary swell $S_2$.
In b) and c), for $W$ and $S$, the period may be taken at this stage as a median period followed by a sensitivity study, or a ±15% plateau as recommended in BV (2004).

Step 2 : “$X$ governed” and “$X$&$Y$ driven” conditions, taking into account the findings of step 1 as to the prevailing parameters and the “most critical” directions, where applicable.

Step 3 : Other combinations in the region(s) of interest, and detailed assessment using the relevant (height-period of swell, $V$-$W$-$T$) contours of all variables. Several sub-steps might be used, as appropriate (see note 2 below).

Note 1 : the “response of interest” is generally not a single response, but several responses (e.g. tension in different lines, stresses at different location in the structure, …), that may have maxima for different combinations of the variables.

Note 2 : In the above, a given step is not intended to provide a single “critical” combination, but only indications, for the next step, on the area around which the maximum response can be expected. Using the polygonal contours will not preclude / might not dispense from a more accurate scanning, but will permit a safer and easier identification of the ‘most critical’ conditions, for which more refined analysis is of interest.

APPLICATION

The contour approach was applied to the mooring system of a typical 2mmbbl FPSO in West Africa, for the evaluation of extreme (100 year Return Period) tensions in mooring lines.
The vessel is spread-moored by 12 lines (chain - wire rope - chain), arranged in four bundles of three lines each. The export lines are also modelled (see figure 11).

A number of sea-state conditions were analysed, with the ARIANE® mooring analysis software, with systematic scanning over selected combinations and sensitivity studies, in order to evaluate and compare the approaches and methodologies presented above. Some interesting conclusions and examples are presented below.
**Directional response**

The study evidenced that the response of a typical spread moored FPSO is governed by the direction of elements, before intensities and other parameters.

The most critical direction of an element (the direction leading to maximum response, e.g. the tension in a given line) might then differ from both the direction of maximum intensity and the direction of maximum response (at a given intensity). Besides, the most critical directions are generally different for each element (swell, wind, wind-sea, and current). As a consequence, the traditional “all-in-line” assumption, besides omitting that the direction of elements are generally independent or weakly correlated, is un-conservative.

On the other hand, it was observed, over a large number of combinations, that the critical direction of one element is almost independent of its return period and of other elements in the combinations, including the case where it is acting alone. This opens to a significantly simplification in the scanning process, using the above defined “X only” cases.

The figure 12 below is showing for illustration the variations with wave incidence and period of tensions (mean tension, low frequency, and quasi-dynamic tension) in one bundle.

**Application of polygonal contours**

Using polygonal contours, i.e. a limited number of combinations of the intensity of elements, was found an efficient method to capture maxima of response, providing a reasonably conservative estimate of the maximum response with respect to the "exact" I-FORM contour. Comparisons were also made with the "traditional" combinations of "all-in-line" elements, with Return Periods of 100 years for all, or 100 and 10 year combinations, as were typically used in the past. Contours gave lower responses showing that, in the analysed cases, the inadequate modelling of intensity correlations in the traditional combination overcome the deficiency in accounting for independence of directions. This may not be the case for another response, and contours results are thus deemed far more reliable.

As an example of the application of polygonal contours, the combinations of wind and wind-sea were analysed, based on the above presented 3D contours, assuming independent directions over a selected sector. Both the polygonal contour and the “single point” simplification were analysed. The I-FORM contour was also studied in the region identified as the most critical by the polygonal contour, for comparison. Maxima of line tensions were taken as the design maximum over 20 simulations, following Bureau Veritas methodology. Results are summarized in the table below, and confirm the validity of simplified contour.

<table>
<thead>
<tr>
<th>Method</th>
<th>Max. Tension in line 1 (kN)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygonal contour</td>
<td>1220</td>
<td>ref</td>
</tr>
<tr>
<td>I-FORM contour</td>
<td>1080</td>
<td>-11 %</td>
</tr>
<tr>
<td>“Single point” comb.</td>
<td>1390</td>
<td>+14 %</td>
</tr>
</tbody>
</table>

![Fig. 13: scanning over the directions of wind and wind sea, for one point of polygonal contour. (Tension as a function of wind direction shown)](image)

![Fig. 14: scanning over polygonal contour (envelope over directions)](image)

**Scanning**

In addition to the above considerations, this work highlighted that an organised step by step scanning through polygonal contours, in conjunction with sensitivity analysis, was an efficient method to identify the “most critical” combinations. A proper understanding of the response of the structure (e.g., for a mooring, the relative contribution to the different terms (mean, low frequency, and quasi-dynamic) of line tension, is however a key factor in this process.
CONCLUSIONS

As a continuation of earlier efforts towards Response Based Design criteria, the I-FORM methodology, derived from structural reliability techniques, is considered to evaluate the extreme response of a structure, under long term (slowly) varying weather conditions, that are described by the statistics of metocean parameters representing the 'metocean events' (short term sea-states) met by the structure.

With I-FORM, a contour (surface) is defined, from which the response corresponding to a specified probability of exceedance can be evaluated as the maximum of the response of interest over the contour, thus turning the reliability problem into a standard scanning problem. As the contour surface is depending only on the joint distribution of above parameters, no prior knowledge of the response is needed to establish the contour.

A first task, requiring a very substantial effort from Oceanographers, is to define the variables and their joint probability distributions. Assessing the correlations between variables is an important step in this process. However, the partitioning of complex sea-states results in complex conditions, difficult to model. This would require further investigations, in relation with the development of the partitioning techniques themselves.

Besides, defining contours for the low probability levels required for designing structures implies an extrapolation of the (joint) distributions including all metocean events. However, different techniques are generally used to derive extreme values of parameters. This is raising theoretical difficulties that would require further work. The inclusion of the short term variability of the response, not addressed here, is indeed a related issue.

For a multivariate environment, the contour is a multivariate hypersurface. Such contour can be built from the unique contour of independent variables and the specific (generally 2D or 3D) contours of correlated variables, and can be scanned by organising the variables in a tree of metocean/response variables.

The approach of "polygonal contours" is proposed, as a practical method to implement a multivariate contour approach using standard engineering tools. It appears as an extension, and a rationalisation in a Response Base context, of the traditional concept of "associated values".

The unique polygonal contour of independent variables is presented. Some typical contours of dependant variables are presented for illustration of the method. A strategy for scanning through polygonal contours is presented, by which, through a step by step approach, the effort to predict a response with a specified return period RP and to identify the most critical combinations of variables (around which maximum response can be expected) can be optimised.

Application to the mooring system of a West Africa FPSO confirmed the feasibility and the adequacy of the proposed methodology, when implemented in an organised way, with a proper understanding of both the metocean conditions and the response being analysed. Whilst initially developed around the specific conditions of West Africa, it is intended that this approach can be translated to other areas, with adjustments to account for the type of metocean conditions.

Considerations of the directionality of metocean effects are both evidence for Oceanographers and, for more than 30 years, the current practice for Designers. Besides the practical difficulties, modelling the direction through a standard Gaussian variable might be inadequate. An approach is proposed by which some (non I-FORM) "contours" can be defined, pending further development in this area.

ACKNOWLEDGEMENTS

The work reported in this paper was prepared within the frame of the joint industry project "Joint Probabilities of Wind/Waves/Current and Response Based Design of FPSO, Mooring and Risers" (JP&RBD) lead by TOTAL, IFREMER, University of Bretagne Sud (UBS), Technical University of Compiègne (UTC), Bureau Veritas (BV), Principia R&D and IFP. The contribution of project Partners is gratefully acknowledged.

The authors wish to thank Block 17 Concessionnaire SONANGOL, Sociedade Nacional de Combustíveis de Angola, EP Sonangol, as well as Total E&P Angola's partners in Block 17 - Esso Exploration Angola (Block 17) Ltd., BP Exploration (Angola) Ltd., Statoil Angola Block 17 A.S., Norsk Hydro - for their authorisation to publish this paper.

The authors also wish to thank their respective Companies for permission to publish this paper. The views expressed are those of the authors, and do not necessarily reflect those of their Companies.

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