ABSTRACT

In order to investigate the local response of a ship structure, it is necessary to transfer the seakeeping loading to a 3DFEM model of the structure. A common approach is to transfer the seakeeping loads calculated by a BEM method to the FEM model. Following the need to take into account the dynamic response of the ship to the wave excitation, some methods based on a modal approach have been recently developed that include the dry structural modes in the hydro-structure coupling procedure and allow to compute the springing and whipping response of the ship structure to the seakeeping loads.

In the context of the fatigue life assessment of a structural detail, a very fine FE model is required. A very large number of seakeeping loading cases also need to be considered to account for all the conditions encountered by the ship through its life. It becomes then clear that because of the CPU time issue, the whole FE model can not be very fine. This is why a hierarchical top-down analysis procedure is commonly used, in which the global ship structure is modelled in a coarse manner using one finite element between web frames. The structural details are modelled separately using a fine meshing. Such top-down methods are commonly used for the estimation of the quasi-static response of structural details to the seakeeping loads.

This paper presents a methodology in which a top-down method is used to estimate the springing response of a ship structural detail loaded with wave pressure, and its fatigue life. The global dry structural modes are transferred to the detail fine model using the shape functions of the finite elements of the global model. The hydrodynamic pressures are computed directly on the fine mesh model, avoiding any interpolation error. The imposed displacements at the fine mesh boundary are computed using the same method that is used to transfer the structural mode shapes, and the local pressure induced loads and inertia loads are applied on the fine mesh nodes.

This method is applied for the calculation of the elongation of a strain gauge which is installed in the passage way of an ultra large container ship.

NOMENCLATURE

ULCS Ultra Large Container Ship
INTRODUCTION

The overall methodology for the transfer of hydroelastic springing loads to a 3D FE model was presented in Malenica (2008) [8]. However, the main focus of that paper was on the global structural response to linear seakeeping loads. The commonly used top-down procedure for the quasi-static case is as follows. The quasi-static response of the global structural model to the seakeeping loads is computed. Then the deformations at the boundaries of the local fine model are interpolated from the global model. The inertia loads corresponding to the 6 rigid body degrees of freedom are then applied globally on the fine model. The incident and diffraction pressure loads are applied on the local fine model if it is located on the wet area of the ship. The radiation and restoring pressures based on the rigid body motions are also applied if necessary. The method that is presented in this paper is a generalization of this procedure to the hydroelastic case, based on a generalized modes approach. The inertia and pressure loads corresponding to the rigid body modes and dry elastic natural modes are directly calculated and applied on the local fine model. The prescribed deformations at the boundary of the local fine model are calculated using projections and the shape functions of the global model elements. The purpose of this paper is to present in detail this generalized top-down procedure for the hydroelastic case. The main original aspects of this procedure are:

- Transfer of the displacements from the global model to the local model using projection and elements shape functions.
- Seakeeping inertia loads at the local model nodes for rigid body and flexible modes.
- Exact seakeeping wave, restoring and radiation loads calculated at the local model nodes for rigid body and flexible modes.

LINEAR HYDROELASTIC MODEL IN FREQUENCY DOMAIN

The general methodology for hydroelastic seakeeping model is rather well known and the first developments can be attributed to Bishop & Price (1979) [1]. In their work they used a Timoshenko beam model as a simplified model of the structure and strip theory for the seakeeping part. Since then several more or less sophisticated models were proposed: Wu & Price (1986) [2], Wu & Moan (1996) [3], Xia & Wang (1997) [4], Korobkin (2005) [5]. Below we give a brief introduction of the basic principles of the model used in this study. The 3DBEM model for the seakeeping is coupled to a 3DFEM model of the ship structure. A more detailed description of the applied 3DBEM model can be found in Newman (1994) [6] and Malenica et al (2003) [7].

Linear hydroelastic Model

In contrast to the well known rigid body seakeeping model, the hydroelastic model basically extends the motion representation with the additional modes of motion/deformation chosen as a series of the dry structural natural modes. We write:

$$\{H\}(x,y,z) = \sum_{i=1}^{N} \xi_i h'(x,y,z)$$  \hspace{1cm} (1)

where $h'(x,y,z)$, denotes the general motion/deformation mode which can be either rigid or elastic. The above decomposition leads to additional radiation boundary value problems (BVP) for elastic modes, with the following change in the body boundary condition:

$$\frac{\partial \phi_R}{\partial n} = \{h\}_j \{n\}$$  \hspace{1cm} (2)

After solving the different BVP-s the resulting pressure is calculated using the Bernoulli equation and integrated over the wet surface in order to obtain the corresponding forces, so that the following coupled dynamic equation can be written:

$$\{-\omega_c^2 (\left[m\right] + \left[A\right]) - i\omega_c (\left[b\right] + \left[B\right]) + \left[c\right] + \left[C\right]\} \{\xi\} = \{F^{DI}\}$$  \hspace{1cm} (3)

where:

$\left[m\right]$ : modal genuine mass matrix  
$\left[A\right]$ : hydrodynamic added mass matrix  
$\left[b\right]$ : structural damping matrix  
$\left[B\right]$ : hydrodynamic damping matrix  
$\left[c\right]$ : modal structural stiffness matrix  
$\left[C\right]$ : hydrostatic stiffness matrix  
$\{\xi\}$ : modal amplitudes  
$\{F^{DI}\}$ : modal excitation

The solution of the above equation gives the motion amplitudes and phase angles by which the problem is formally solved. Note that the motion equation includes both 6 rigid body modes and a certain number of elastic modes. Several technical difficulties need to be solved before arriving to the above motion (Eq. 3). Certainly the most difficult one is the solution of the corresponding hydrodynamic BVP. In this paper we do not enter into the detailed description of the methods used to solve the seakeeping problem at forward speed and we just mention that these difficulties remain the same, both for the rigid and elastic body. It is fair to say that the numerical methods which are used
to solve the seakeeping problem nowadays are not fully ready yet for a general combination of speed, heading and frequency. However, most of the methods have approximate solutions to account for the forward velocity. The method used in this paper is the so-called encounter frequency approximation which was reasonably well validated for the rigid body case.

**Springing**

The solution of the hydroelastic motion equation (3) includes the linear springing response automatically. However, there is no clear justification for considering springing as a linear phenomenon, at least as far as the wave excitation part is considered. Indeed, the experience shows that many type of ships suffer from more or less important structural response around their first structural natural frequencies even if there is no important wave energy around these frequencies. The existence of such structural response can only be explained by introducing nonlinearities into the hydrodynamic model. The well-known example is the springing of the tendons of TLPs where the hydrodynamic excitation was successfully explained using the weakly non-linear second order theory. In principle similar kind of methods should also be applied in the case of ships, but unfortunately the state of the art in ship hydrodynamics does not allow for that so that only approximate and empirical methods can be used. However, due to their huge dimensions and particular structural properties, the case of ULCS is quite particular because their first structural natural frequencies become quite low so that they can be excited in a linear sense. This means that the linearly excited springing might become dominant which justifies the use of a linear model for its assessment.

**MASS PROPERTIES OF THE GLOBAL FE MODEL**

In order to solve the motions equation (Eq. 3), one needs to estimate the $N \times N$ modal mass matrix of the global FE model. Where $N$ is the total number of modes, rigid and dry flexible modes. Since the structural response to the seakeeping loads will be computed using a FE solver, the modal mass matrix that will be used needs to be consistent with this particular FE solver.

The modal mass matrix of the ship could easily be calculated using the $6 \times 6$ rigid body mass matrix for the rigid modes part and the modal mass of each elastic mode for the diagonal terms of the elastic modes mass. We would then have:

$$[m] = \begin{bmatrix} m_r & 0 \\ 0 & m_f \end{bmatrix}$$  (4)

where:

$m_r : 6 \times 6$ rigid body modes mass matrix

$m_f : N \times N$ diagonal flexible modes mass matrix with $N$ being the number of flexible modes

However, since we want to apply the inertia loads corresponding to the seakeeping modal accelerations on the FE model nodes, we need to calculate the inertia forces coefficients at each node for each mode. We will then use them to compute the global ship modal mass matrix.

At each node of the global FE model, the mode shapes matrix is written as:

$$\begin{pmatrix} h_1^r \\ h_1^f \\ \vdots \\ h_N^r \\ h_N^f \end{pmatrix}$$

where:

$N$: total number of modes
$h_i^r$: local translation along x axis for mode $i$
$h_i^f$: local translation along y axis for mode $i$
$h_i^z$: local translation along z axis for mode $i$
$h_i^\phi$: local rotation around x axis for mode $i$
$h_i^\theta$: local rotation around y axis for mode $i$
$h_i^\psi$: local rotation around z axis for mode $i$

The rigid body modes can be defined with respect with any arbitrary point. However the center of gravity of the structure is often used. For each rigid body mode, in order to calculate the mass and inertia associated to each node by the FE solver, a unit global acceleration load case is created with respect to an arbitrary reference point. The forces applied by the FE solver for each of those unit acceleration cases are transported from the reference point to the local coordinates system of each node. This gives us the $6 \times 6$ local mass matrix at each node for the rigid body modes.

The most straightforward approach would then be to compute the global modal mass matrix with:

$$[m] = \sum_{i=1}^{N_n} [h_i]^T [M_i] [h_i]$$  (6)

where:

$N_n$: number of nodes in the global FE model
$[h_i]$: mode shapes matrix at node $i$
$[M_i]$: local $6 \times 6$ rigid body mass matrix at node $i$
However the finite element modelling of a ship structure can lead to inertia loads on a node which do no only depend on the motions of this particular node. For example when interpolation elements are used to spread a concentrated mass over a set of nodes, the inertia loads on a particular node of this set will indirectly depend on the motions of all other nodes of the set. This is why the nodal masses for the elastic modes are needed and can not be estimated only from the rigid body mass and the flexible mode shapes.

The forces on each node for each dry natural modes are used to build the $6 \times N$ nodal mass matrix at each node which can be written as:

$$[M] = \begin{pmatrix}
m_{1x} & \ldots & m_{i_1x} & \ldots & m_{Nx} \\
m_{1y} & \ldots & m_{i_1y} & \ldots & m_{Ny} \\
m_{1z} & \ldots & m_{i_1z} & \ldots & m_{Nz} \\
m_{1\phi} & \ldots & m_{i_1\phi} & \ldots & m_{N\phi} \\
m_{1\theta} & \ldots & m_{i_1\theta} & \ldots & m_{N\theta} \\
m_{1\psi} & \ldots & m_{i_1\psi} & \ldots & m_{N\psi}
\end{pmatrix}$$

(7)

where:

$N$ : total number of modes
$m_{i_1}^x$ : force along x axis for a unit acceleration along mode $i$
$m_{i_1}^y$ : force along y axis for a unit acceleration along mode $i$
$m_{i_1}^z$ : force along z axis for a unit acceleration along mode $i$
$m_{i_1}^\phi$ : moment around x axis for a unit acceleration along mode $i$
$m_{i_1}^\theta$ : moment around y axis for a unit acceleration along mode $i$
$m_{i_1}^\psi$ : moment around z axis for a unit acceleration along mode $i$

Finally the global ship $N \times N$ mass matrix is built using a sum over the FE model nodes:

$$[m] = \sum_{i=1}^{N_n} [h_i]^T [M_i]$$

(8)

where:

$N_n$ : number of nodes in the global FE model
$[h_i]$ : mode shapes matrix at node $i$
$[M_i]$ : local $N \times N$ modal mass matrix at node $i$

Most FE solvers scale the natural modes in order to get a unit modal mass. For the flexible modes, we check that the diagonal terms of the mass matrix that we calculate are equal to the modal masses computed by the FE solver.
Transfer of mode shapes on the detail interface nodes

The mode shapes are transferred from the global FE model to the interface nodes of the structural detail FE model. The procedure used for this transfer is shown on figure 2. For each node of the detail, the closest element of the global FE model is found based on projections on the surface, edges and nodes of the elements. An optimized algorithm using the distance to the elements center and the distance based on projections is used. Once the closest element has been found, the detail node is projected on it and its local coordinates are computed. Based on the local coordinates of the detail node on the element, the mode shapes at the element nodes and the shape functions of the element, the mode shapes matrix for a given elastic mode at the detail node is calculated using:

\[ M_f = \sum_{i=1}^{N_n} N_i(\xi, \eta)[h_i] \]  

where:
- \( N_n \): total number of nodes in the global FE model element
- \( \xi, \eta \): local coordinates of the detail node in the global FE model element
- \( N_i \): shape function of the global FE model element for node \( i \)
- \( [h_i] \): mode shapes matrix at node \( i \) of the global FE model element

Imposed displacements FE analysis

Once the mode shapes at the interface nodes are approximated, the imposed displacement FE analysis gives, for each elastic mode, the mode shapes at all nodes of the structural detail (see figure 3).

Nodal modal mass

The nodal masses for the rigid body modes are obtained in the exact same way as it is done for the global model. For the flexible modes, the nodal masses for the rigid modes and the mode shapes are used in order to approximate the nodal masses at each detail node with:

\[ [M_f] = [M_r][h_f] \]  

where:
- \( [M_f] \): flexible modes part of the nodal mass matrix with \( N_f \) being the number of flexible modes
- \( [M_r] \): 6 x 6 rigid modes part of the nodal mass matrix
- \( [h_f] \): flexible modes part of the mode shapes matrix

The complete mass matrix at each node is then:

\[ [M] = [M_r][M_f] \]  

SEAKEEPING LOADS ON THE LOCAL STRUCTURAL DETAIL

The total seakeeping loads, including hydrodynamic pressures and inertia, are transferred to the global FE model using the method described in Malenica (2008) [8]. The motion equation 3 is solved and the structural response to the seakeeping loads is calculated as in [8]. The local structural model needs to be loaded in a consistent way, based on the wave excitation and the global ship response.

Hydrodynamic pressure loads

A gauss points integration scheme is used to apply the pressure loads on the structural detail as nodal forces. The pressures are computed at the structural gauss points of the detail in the same way they are computed on the ship hull. The recalculation
of the hydrodynamic pressure on the structural gauss points, is possible thanks to the particularities of the BIE method which gives a continuous representation of the potential through the whole fluid domain $Z < 0$. In this way the communication between the hydrodynamic and structural codes is extremely simplified. Indeed, it is enough for the structural code to give the coordinates of the points where the potential is required and the hydrodynamic code just evaluates the corresponding potential by:

$$\varphi(x_s) = \int\int_{\mathcal{S}} \sigma(x_h) G(x_h, x_s) dS$$

(12)

where $x_s = (x_s, y_s, z_s)$ denotes the structural point and $x_h = (x_h, y_h, z_h)$ the hydrodynamic point.

The total pressure at each gauss point is then calculated with:

$$p_{tot} = p_{inc} + p_{diff} + \sum_{i=1}^{N} \xi_i (p_i^r + p_i^{rst})$$

(13)

where:

- $p_{inc}$: incident pressure
- $p_{diff}$: diffraction pressure
- $p_i^r$: radiation pressure coefficient for mode $i$
- $p_i^{rst}$: restoring pressure coefficient for mode $i$
- $\xi_i$: modal motions response for mode $i$
- $N$: number of modes

**Inertia Loads**

The inertia loads at each node of the detail are calculated in a straight forward manner from the modal motions response and the nodal mass matrix as follows:

$$\{F_i\} = [M]\{\ddot{\xi}_i\}$$

(14)

where:

- $[M]$: nodal mass matrix
- $\{\ddot{\xi}_i\}$: modal acceleration response vector

Note that here the nodal mass matrix (obtained as in equation 11) is a $6 \times N$ matrix where $N$ is the number of rigid body plus flexible modes.

**Imposed displacements**

The displacements obtained from the global structural response are transferred to the interface nodes of the detail in the exact same way as the flexible mode shapes were transferred and at each interface node, the displacement is obtained by:

$$\{d\} = \sum_{i=1}^{N_n} N_i(\xi, \eta) \{d_i\}$$

(15)

where:

- $N_n$: total number of nodes in the closest global FE model element
- $\xi, \eta$: local coordinates of the detail node in the closest global FE model element
- $N_i$: shape function of the closest global FE model element for node $i$
- $\{d_i\}$: structural response displacements at node $i$ of the closest global FE model element

**VALIDATION OF THE METHOD**

In order to validate the method, we extract the area of the structural detail from the global FE model and use it in the top down procedure (figure 4). Then we can compare the results we get on the extracted detail with the ones we get directly on the global model. We use the elongation of a rod element located in the passage way for it represents an actual strain gauge which is installed on the ship for full scale measurements. The difference in the calculated elongation of this gauge for each mode on the global model and on the extracted detail can be seen on figure 5.

![Figure 3. Flexible mode shape on the structural detail.](image-url)
For the first three dry flexible natural modes, the difference in the modal elongation of the strain gauge is less than one percent. Some of the higher modes (8 and 9) show a larger difference, those are flexible modes involving large local deformation of the longitudinal bulkheads compared to the global structure deformations. Keeping those modes or not could be discussed. Only the first three modes will be significantly excited by the linear seakeeping loads because their natural frequency is within the wave encounter frequency range. Indeed, the natural frequency of the higher modes is above 4.5 radians per second which is far out of the wave encounter frequency range. Consequently we can conclude that, in terms of modal deformations and for dry flexible natural modes involving mainly global structural deformations, the procedure presented in this paper for relatively small details is very accurate. The strain gauge elongation RAO in head sea can be seen on figure 6. The figure shows the RAO obtained on the global model and on the local detail directly extracted from it. The difference is less than one tenth of a percent for all wave encounter frequencies, this perfect fit validates the top-down procedure presented here, under the assumption of a relatively small detail.

**NUMERICAL RESULTS**

The figure 7 shows the strain gauge elongation RAO calculated on the local fine model. The quasi-static and hydroelastic contributions to the total response are displayed. The hydroelastic effects clearly become dominant for high wave encounter frequencies, mainly close to the first vertical bending mode resonance. Therefore it is crucial to take those effects into account for such types of ships.

The figure 8, 9 and 10 show the strain gauge elongation RAO calculated on the global model and on the local fine model for various wave incidences. It appears that in head and quartering seas, there is no significant difference in the elongation calculated on the global model and on the local fine model. However, a slight difference can be seen in beam sea. At the strain gauge location, a relatively coarse model is sufficient to capture the elongation of the gauge due to global vertical bending of the structure. This is why no significant difference is observed in those cases. In beam sea, the global torsion of the structure is
CONCLUSIONS

A consistent and robust top-down procedure has been presented. This procedure is applied to estimate the hydroelastic response of a ship structural detail. The method has been validated by comparing the structural response obtained directly on the global FE model, and on a local portion of it, on which the top down procedure is applied. A local fine mesh model of a structural detail has been build in order to estimate the elongation of a strain gauge which was installed in the passage way of an ULCS for full scale measurements. The hydroelastic effects become dominant at high wave encounter frequency and can have a significant influence on the fatigue life of some structural details. The numerical results show that significant differences in the estimated elongation on the global and local models only occur in beam sea. The fine mesh is necessary to capture properly the local structural response to torsion. This top-down procedure, including hydroelastic effects will allow for a consistent comparison of the numerical results with full scale measurements in order to validate the complete hydro-structure coupling procedure.
REFERENCES


