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Impact of the use of *FullQTF* on LNGC Moored in Shallow Water Studies

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Abstract

The prediction of slow drift motions for the design of a mooring system is usually made using the *Newman* approximation [1], based on the assumption of a very low resonance frequency of the system and small contribution of the second order wave fields. This hypothesis is commonly satisfied for most parts of the mooring systems in deep water. However, this is not the case for LNG terminals moored in shallow water.

Unlike the *Newman* approximation, the *FullQTF* formulation to compute the low frequency wave loads is more accurate but requires much longer time of computation, which presents limitations in practice when a large quantity of simulations is needed.

Further to the work presented in [2] on the quadratic transfer function (QTF) of low-frequency loading, a new approximation has been developed in [3]. The *F1* approximation gives comparable results to the *FullQTF* and presents the interesting aspect that the loads time series can be reconstructed by means of simple summations, presenting the same efficiency in computation time as *Newman* approximation.

In this paper, main parameters of mooring systems are analyzed to evaluate the impact of the choice of each method, *Newman*, *F1* or *FullQTF*. Indeed, this choice is a compromise between calculation time and accuracy of results. The conclusions raised are underlined in the study of an LNG terminal.

Introduction

More and more LNG carriers are built across the world to transport the liquefied gas from a location to another. These locations sometimes propose LNG terminals in the proximity of the harbors, thus in shallow water. For those terminals, the accurate prediction of low-frequency wave loads is a key issue in the design of mooring systems.

The approximation proposed by *Newman* (1974) is generally used for the simulations of low frequency loading due to the fact that it is based only on the diagonal terms of the QTF (Quadratic Transfer Functions). Those terms, the mean drift loads, are contributed only by the first-order wave field and body motions and are easily calculated once the first order solution is obtained. In moderate and deep water depths it can be shown that the second order wave field contribution is small comparing to the first order one. This way, the assumptions applied to the *Newman* approximation are satisfied and the results obtained through this approximation can be considered adequate. However, this is not the case in shallow water, where the contribution of the second order wave field can not be neglected.

The use of the complete QTF matrix gives more accurate estimations of the low-frequency loads, but it requires the solution of the second order problem and the time series reconstruction is more time-demanding. Recently, a new approximation has been presented in [2] and [3] considering that the resonant frequencies of mooring systems are often small (<<1). This approximation consists in developing the QTF as an expansion of difference frequency $\Delta\omega$ and keeping the terms dependent on the first order quantities T^0 and another term T^1 linearly proportional to $\Delta\omega$. From this expansion two approximations can be derived: the *F0* approximation, which is equivalent to the *Newman* approximation, and the *F1* approximation.

In this paper the *F0* (*Newman*) and *F1* approximations are compared to the *FullQTF* considering the influence of the water depth and mooring stiffness. The time efficiency of each method is also addressed.

Finally, the conclusions raised are highlighted in an LNG terminal example, under specific environmental conditions.

QTF Formulations

The full quadratic transfer function (QTF) of low-frequency loads is composed of two parts: one depends on the quadratic product of the first-order quantities and another is contributed by the second-order potentials:

$$T(\omega_1,\omega_2) = T_1(\omega_1,\omega_2) + T_2(\omega_1,\omega_2) \qquad \textbf{(1)}$$

The *Newman* approximation is obtained by disregarding the second term of equation (1). This approximation is based on the assumption that $\Omega \to 0$, where $\Omega = \omega_1 - \omega_2$. For shallow water applications, however, the second-order potentials contributions can not be neglected and the *Newman* approximation may largely underestimate the low frequency loading.

Recently, a new approximation has been presented in [4]. Assuming $\Omega << 1$, the QTF is developed as an expansion:

$$T(\omega_1,\omega_2) = T^0(\omega,\omega) + i\,\Omega\, T^1(\omega,\omega) + \mathrm{O}(\Omega^2) \qquad \textbf{(2)}$$

The quadratic transfer function $T(\omega_1,\omega_2)$ is then composed of one component $T^0(\omega)$ depending on $\omega = (\omega_1 + \omega_2)/2$ and another $\Omega\, T^1(\omega)$ linearly proportional to $\Omega = \omega_1 - \omega_2$. From this expansion two approximations are derived. The *F0 approximation*, which is equivalent to the *Newman* approximation, is based only on the use of $T^0(\omega)$ so that is of $O(1)$ and the *F1 approximation* that consider also the term $\Omega\, T^1(\omega)$ so that is of $O(\Omega)$.

Considering the fact that:

$$T^{0/1}(\omega,\omega) = \frac{1}{2}\left[T^{0/1}(\omega_i,\omega_i) + T^{0/1}(\omega_j,\omega_j)\right] + \mathrm{O}\left[\Omega^2\right] \quad \text{with} \quad \omega = \frac{\omega_i + \omega_j}{2} \qquad \textbf{(3)}$$

The *F1 approximation* in [3] can be rewritten as:

$$T(\omega_i,\omega_j) = \frac{1}{2}\left[T^0(\omega_i,\omega_i) + T^0(\omega_j,\omega_j)\right] + i\frac{1}{2}\left[(\omega_i - \omega_j)\cdot\left(T^1(\omega_i,\omega_i) + T^1(\omega_j,\omega_j)\right)\right] \qquad \textbf{(4)}$$

Time series reconstructions

For an irregular sea, the wave elevation can be noted as follows:

$$\eta(t)=\Re\left\{\sum_{j=1}^{N} a_j e^{i(\omega t-k\cdot x+\varphi)}\right\} \text{ with } a_j=\sqrt{2.\cdot S(\omega_j)\cdot d\omega_j} \tag{5}$$

Where $S(\omega)$ is the irregular wave spectral density and $d\omega$ is the sampling space of the spectrum.
The low frequency loads can now be defined in time domain as below.

FullQTF formulation

FullQTF is a two dimensions complex matrix. The Real part of the matrix is symmetrical whereas the Imaginary one is anti-symmetrical. The low-frequency drift loads time signal using the *FullQTF* formulation is obtained by a double summation:

$$F_{FullQTF}(t)=\Re\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\left[T(\omega_i,\omega_j)\cdot a_i.a_j.e^{i\left((\omega_j-\omega_i)t-(k_j-k_i).x+(\varphi_j-\varphi_i)\right)}\right]\right\} \tag{6}$$

F0 approximation (Newman)

This method consists in approximating $|T(\omega_i, \omega_j)|^2$ by a function of $T(\omega_i, \omega_i)$ and $T(\omega_j, \omega_j)$. Thus, the *FullQTF* double summation can be transformed into the simple following formulation:

$$F_0(t)=\Re\left\{\sum_{j=1}^{N} T^0(\omega_j,\omega_j)\cdot a_j.e^{-i\cdot(\omega_j\cdot t-k_j\cdot x+\varphi_j)}\cdot\sum_{j=1}^{N} a_j.e^{i\cdot(\omega_j\cdot t-k_j\cdot x+\varphi_j)}\right\} \tag{7}$$

F1 approximation

Introducing (4) into (6), the time series reconstruction for the *F1* approximation presented in [3] can be obtained as follow:

$$F_1(t)=F_0(t)-\frac{d}{dt}\Re\left\{\sum_{j=1}^{N} T^1(\omega_j,\omega_j)\cdot a_j.e^{-i\cdot(\omega_j\cdot t-k_j\cdot x+\varphi_j)}\cdot\sum_{j=1}^{N} a_j.e^{i\cdot(\omega_j\cdot t-k_j\cdot x+\varphi_j)}\right\} \tag{8}$$

Remarks

The *FullQTF* formulation is the most accurate method to describe the physical phenomenon. However this kind of calculation is very time consuming due to the double summation on the frequencies and the calculation of the *FullQTF* matrix. The *F1* approximation is a new method that provides more accurate results comparing to the *F0* approximation and presents the advantage that the time series reconstruction can be described as simple summations instead of double summations, hence faster to compute.

Numerical results

Different parameter influences are studied before computing a complete example of LNG terminal. Thus, varying water depth, wave period and mooring stiffness, differences between the three methods are evaluated in terms of accuracy and time efficiency.

Influence of water depth and peak period of sea state

In this case, numerical calculations are performed for a standard 138km^3 LNG vessel with main dimensions (Length, Breadth and Draft) = (274m, 44.2m and 11m), respectively. The half hull mesh is composed of 2204 panels.

At a first stage, the vessel is considered fixed and the low-frequency loads are calculated in frequency domain for different water-depths and for sea states with Hs=2.0m and peak periods varying from 6s to 18s.

Figures 1 to 6 present the spectral density of low-frequency loads. For each peak period the left bar is used for *F0* approximation, the middle bar for *F1* approximation and the right bar for the *FullQTF*. Two differences of frequencies (Ω) are evaluated, 0.025 rd/s and 0.05 rd/s, which are directly linked to the resonance of the mooring system. It can be noticed that the *F0* approximation largely underestimates the loads in shallow water comparing to the *FullQTF*. For moderate water depth, such like 60m, the *F0* approximation may give conservative results except for very long waves. In all the cases the *F1* approximation presents results which are comparable to those obtained through the *FullQTF*.

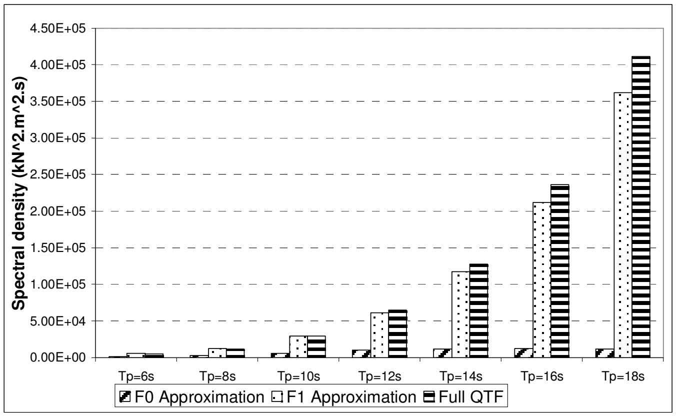

Figure 1 – LF spectral density – WD 15m – Tn ~251s

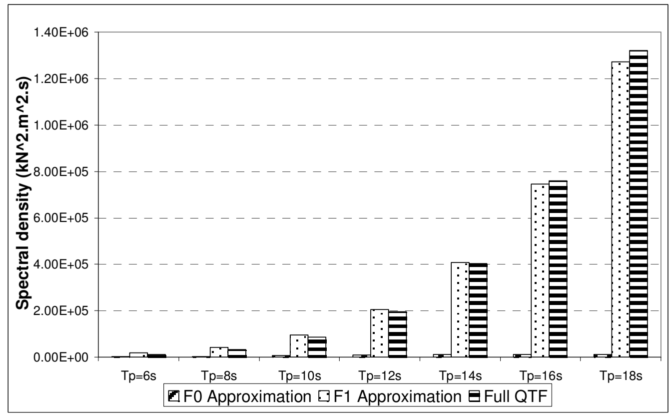

Figure 2 - LF spectral density – WD 15m – Tn ~126s

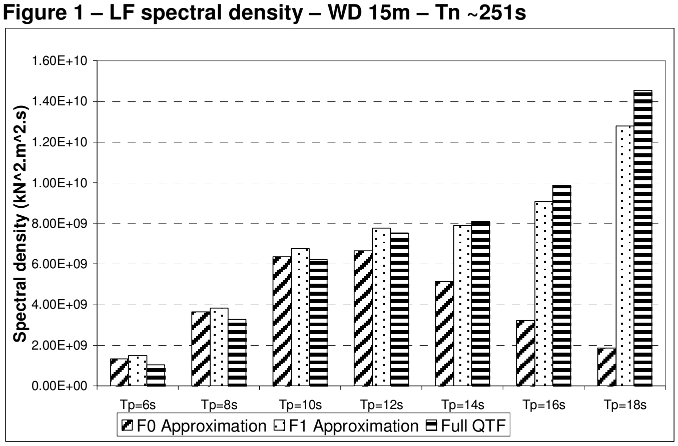

Figure 3 - LF spectral density – WD 30m – Tn ~251s

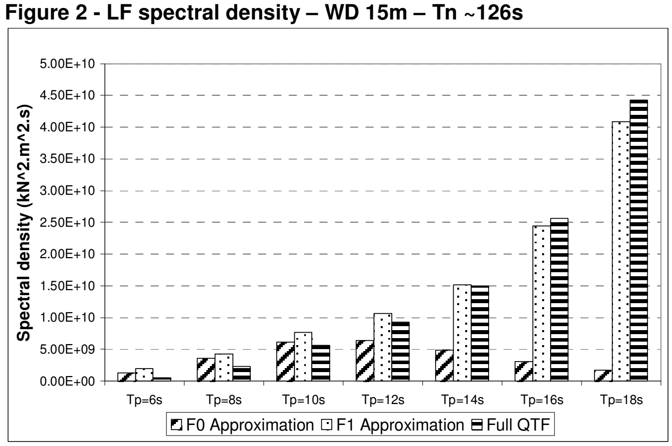

Figure 4 - LF spectral density – WD 30m – Tn ~126s

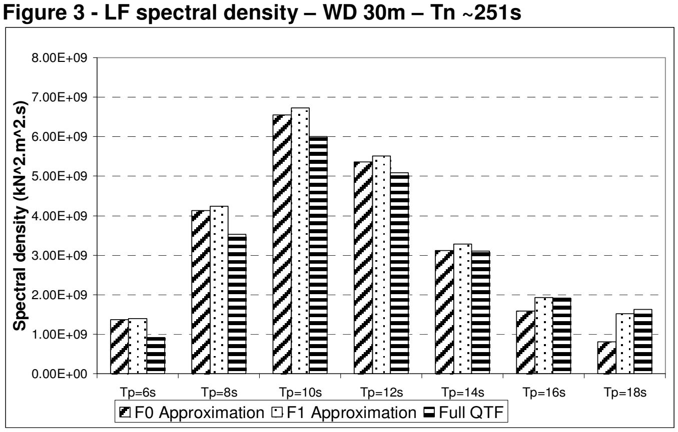

Figure 5 - LF spectral density – WD 60m – Tn ~251s

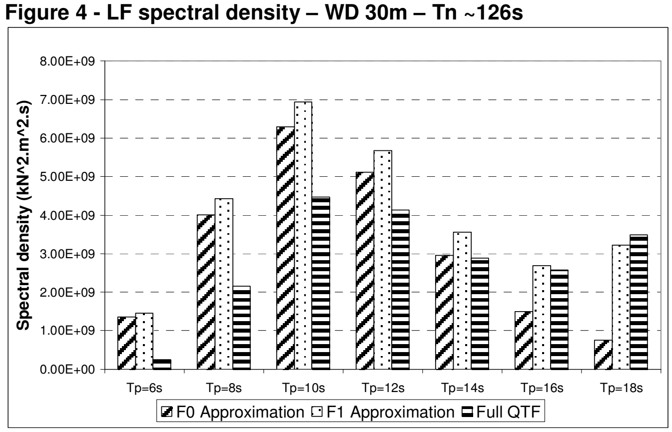

Figure 6 - LF spectral density – WD 60m – Tn ~126s

Regarding the two resonance periods studied from Figure 1 to Figure 6, it is shown that the differences between the three methods increase for smaller resonance periods. To highlight this phenomenon, the mooring stiffness influence is studied in time domain.

Mooring stiffness influence

LNG carrier motions are analyzed using different mooring system stiffnesses. To study the mooring stiffness influence, a typical reference characteristic shown in Figure 7 is multiplied by coefficients from 0.2 to 2.0 with a step of 0.2.

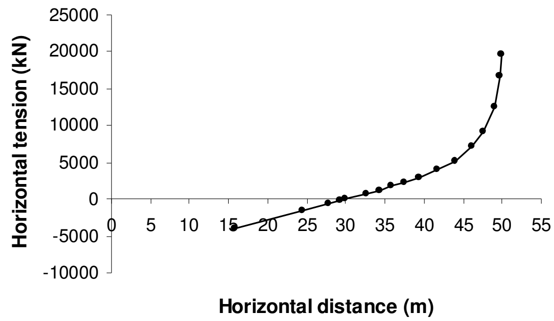

Figure 7: Mooring system characteristic

In each study case, the same environment is imposed. It is composed of waves (Jonswap spectrum with Hs=3m, Tp=10s and γ=3.3). Only surge motion direction has been studied for head sea condition.

Figure 8 and Figure 9 give a comparison of the vessel motions time series for three different mooring stiffnesses, (from the softest on the left to the stiffest on the right), comparing respectively *F0*/FullQTF and *F1*/FullQTF.

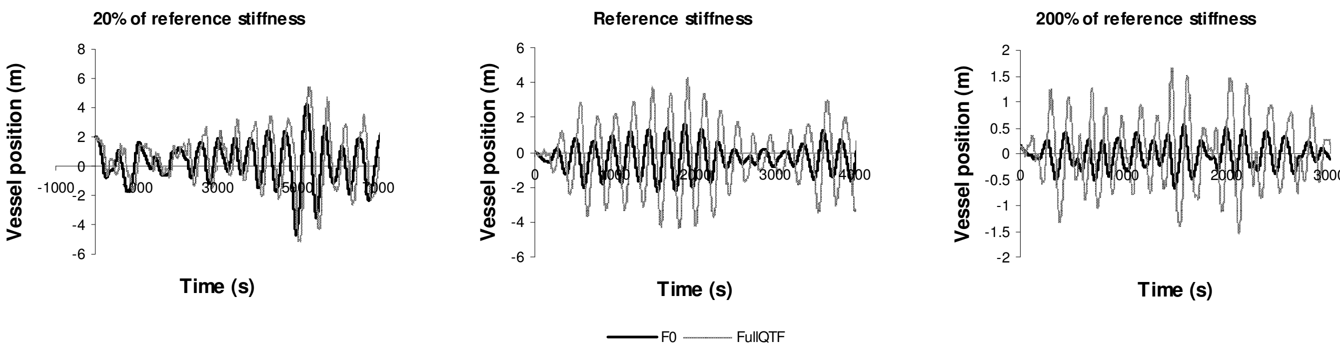

Figure 8: Vessel position time series under *F0* and *FullQTF* drift loads

With reference to Figure 8, it can be clearly shown that changing the global mooring stiffness accentuates the differences between the *F0* and *FullQTF* calculations. Indeed, when the mooring system is very soft, the two resulting signals from *F0* and *FullQTF* are quite similar. On the contrary, when the stiffness is increasing, the time series become more and more different.

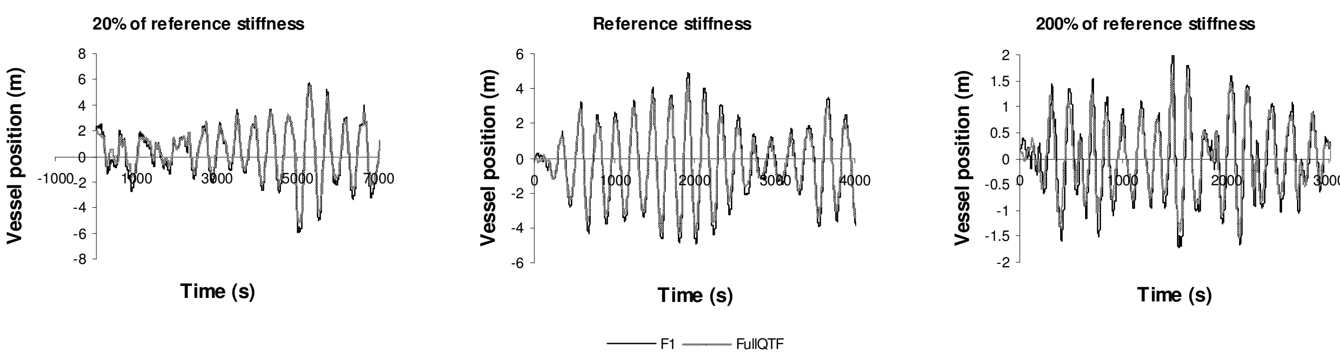

Figure 9: Vessel position time series under *F1* and *FullQTF* drift loads

Figure 9 shows that the differences in the vessel motions time series when using the *F1* approximation or *FullQTF* formulation are not as consequent as when using the *F0* approximation. Indeed, it seems to suggest that the *F1* expression gives suitable results even for the stiffest mooring stiffness evaluated.

In case the *F0* approximation is used with a very soft mooring system, the ratio between the *F0* and *FullQTF* time series standard deviation of loads is about 0.8 whereas it could be less than 0.4 for a stiff mooring system (see Figure 10). In fact, the stiffer is the mooring system, the bigger the difference between the *F0* and *FullQTF* time series.

However the ratio between *F1* and *FullQTF* time series standard deviation is less than 1.10 for most of the mooring stiffnesses, except for the stiffest mooring system with a maximum of 1.25 (see Figure 10). In addition, it can be seen that the ratio is always greater than 1.00, which seems to suggest that the *F1* approximation is conservative.

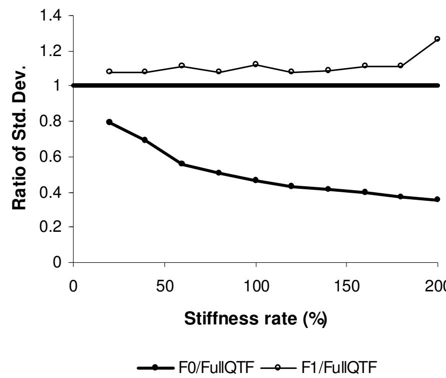

Figure 10: *F0* and *FullQTF* time series standard deviation relative difference function of the stiffness rate

Computation efficiency in time domain reconstruction

As shown in equations (6), (7) and (8), the *FullQTF* reconstruction is based on a double summation on frequencies whereas *F0* and *F1* are based on simple summations. Figure 11 shows an example of comparison between the methods when irregular waves are reconstructed by 200 elementary airy waves.

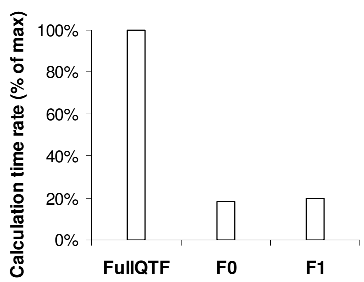

Figure 11: Calculation time comparison between *FullQTF*, *F0* and *F1*

A solution to make *FullQTF* calculations faster is to reduce the number of terms of the "double summation". Thus, a new parameter called $\Omega_{\max}$ is introduced, considering that the vessel is less sensitive to the difference frequency greater than $\Omega_{\max}$. Only terms of the *FullQTF* matrix that satisfy the following inequality are taken into account:

$$\omega_i - \omega_j < \Omega_{\max} \tag{9}$$

Remark: if $\Omega_{max} > \omega_N - \omega_0$, the complete drift load is calculated whereas if $\Omega_{max} = 0.$ only the mean drift load is computed. Figure 12 shows that the computation time decreases significantly when $\Omega_{max} < 0.4$.

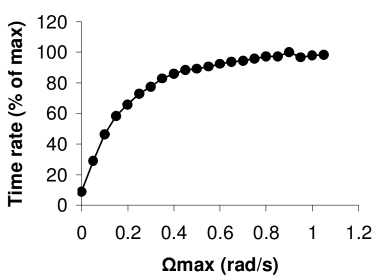

Figure 12: Double summation time consumption in function of $\Omega_{\max}$

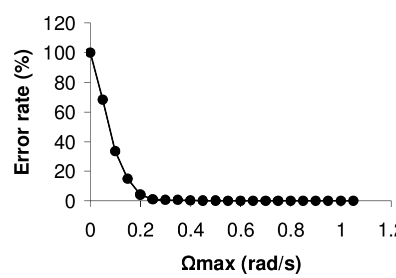

Figure 13: F(t) standard deviations error rate

Choosing $\Omega_{max} = 0.$, there is no more time dependency, meaning that the time series is a constant signal, and standard deviation is logically 100%. In Figure 13, it is possible to see that the wave drift load time series is well recomposed for $\Omega_{\max} > 0.3\ rad/s$, that gives about 20% reduction of the calculation time.

This conclusion is set for this particular example. The main notion is that it is possible to analyze a mooring system and quickly determine a $\Omega_{\max}$ in function of its resonance frequency.

Study case: LNG terminal in shallow water

In this section, one practical example is used to compare the different low frequency wave loads calculation methods in order to illustrate the results previously obtained.

The selected mooring system is a typical configuration for LNG terminals in shallow water. This system, called Soft Yoke (SY) is composed of a fixed structure, mooring legs and the yoke (see Figure 14)[5].

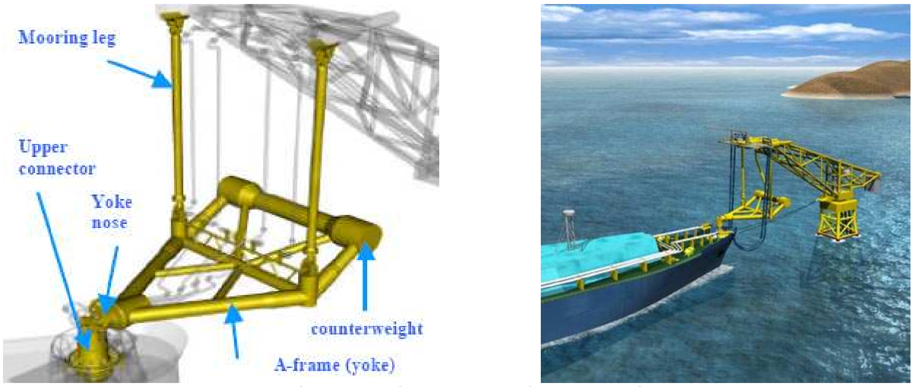

Figure 14: SY for LNG terminals (Cortesy of SBM)

The mooring system is modeled by its characteristic horizontal distance/horizontal tension used as the reference one for the stiffness influence study (see Figure 7). The neutral point is 30 meters from the yoke nose. The mooring system is symmetrical. The LNG carrier can turn around the fixed structure and around the yoke nose. As previously presented, the main properties of the 138km^3 LNG carrier are respectively (Length, Breadth and Draft) = (274m, 44.2m and 11m). The water depth is 15 meters and studies are done with 3 hours time series of vessel motions and SY tensions.

In order to be able to compare correctly the different temporal signals, the same wave spectrum is imposed. The selected spectrum is a Jonswap one defined by respectively (Hs, Tp, γ) = (3m, 10s, 3.3). The wave spectrum is discretized into two hundred elementary Airy waves.

A current load is applied in order to change the vessel azimuth with respect to the wave. Different incidences are studied, thus different relative wave headings. Current is supposed to be constant with a velocity of 1.2m.s^{-1}.

Table 1: Simulation results under different environmental conditions

		Wave: 0°			Wave: 0°, current: 30°			Wave: 0°, current: 60°			Wave: 295°, current: 90°		
		F0	F1	FullQTF	F0	F1	FullQTF	F0	F1	FullQTF	F0	F1	FullQTF
Wave relative heading at mean position (deg)		0.0			17.0			28.5			42.1		
Vessel position	Azimuth std. dev. (deg)	0.00	0.00	0.00	2.88	2.85	3.16	1.47	1.65	1.69	2.27	2.45	2.50
	North std. dev. (m)	0.63	2.19	2.03	3.58	3.67	4.39	4.74	5.42	5.48	6.83	7.40	7.64
	East std. dev. (m)	0.00	0.00	0.00	2.35	2.43	2.44	3.01	3.18	3.19	3.52	3.85	4.01
SY horizontal tension	Std. Dev. (kN)	167.37	602.91	555.28	125.40	412.72	359.72	185.95	397.28	456.42	323.93	574.88	660.10
	Relative diff. / FullQTF	0.30	1.09		0.35	1.15		0.41	0.87		0.49	0.87	

Table 1 illustrates the differences between the drift loads calculation methods varying the relative heading between the wave and the vessel. The first point that could be underlined is that for all simulations, *F0* approximation underestimates the loads on the mooring system. Indeed, this difference is always over 50% on the standard deviation of loads and is more important in head sea (70%).

Moreover, the *F1* approximation presents comparable results to the FullQTF ones. The maximum difference observed is 15%, and *F1* could underestimate the loads as well as overestimating them.

Conclusions

This paper presents a comparison between different formulations for the calculation of slow drift loads for the applications in shallow water. It has been shown that the *F0* approximation, commonly used in practice, largely underestimates the low-frequency loads for shallow water depths (below 60m) or when the mooring system is stiffened.

The *FullQTF* method, in spite of giving more accurate results, is less efficient in computation time due to the fact that the time reconstruction of the loads is made by double summations instead of simple summations as in the case of *F0* approximation. The *F1* approximation is a new method to evaluate the low-frequency loads in a more accurate way than *F0* approximation. The most interesting aspect of this approximation is that the time reconstruction can be done by simple summations, thus it presents the same calculation time efficiency as the *F0* formulation.

Acknowledgments

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Nomenclature

SY = Soft Yoke
QTF = Quadratic Transfer Function
LNG = Liquefied Natural Gas

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Tables

Figures