HYDROELASTIC ASPECTS OF LARGE CONTAINER SHIPS

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ABSTRACT
The importance of hydroelastic analysis of large and flexible container ships of today is pointed out. A methodology for investigation of this challenging phenomenon is drawn up and a mathematical model is worked out. It includes definition of ship geometry, mass distribution, structure stiffness, and combines ship hydrostatics, hydrodynamics, wave load, ship motion and vibrations. Based on the presented theory, a computer program is developed and applied for hydroelastic analysis of a flexible segmented barge for which model test results of motion and distortion in waves have been available. A correlation analysis of numerical simulation and measured response shows quite good agreement of the transfer functions for heave, pitch, roll, vertical and horizontal bending and torsion. The developed tool is furthermore used for hydroelastic analysis of a large container ship.

1. INTRODUCTION
Large container ships are very flexible and their structural natural frequencies can fall into the range of the encounter frequencies in an ordinary sea spectrum. Therefore, the wave induced hydroelastic response becomes an important issue especially for improving the classification rules and ensuring ship safety. For container ships, due to open cross-section, the lowest elastic natural modes are those of coupled horizontal and torsional vibrations. This coupling is highly pronounced due to the fact that the shear (torsional) centre is below the keel, i.e. far from the centre of gravity. The classical approach to determine ship motions and wave loads is based on the assumption that the ship hull acts as a rigid body [1]. That approach is not reliable enough for ultra large ships due to mutual influence of the wave load and structure response. Therefore, a reliable solution requires analysis of wave load and ship vibrations (springing and whipping) as a coupled hydroelastic problem.

The hydroelastic theory has been established by Bishop and Price as the unified strip-beam theory [2]. The hull model is represented by the Timoshenko beam and the fluid action by the strip theory [3]. The hydroelastic theory is extended to general floating structures in such a way that the space structural modes are obtained by the finite element method and the fluid action is determined by a three-dimensional potential theory [4].

One of the most recent detailed investigations of ship hydroelasticity in case of bulk carrier is elaborated in [5]. Timoshenko beam idealization and 3D FEM model combined with potential flow theory are applied. The vertical bending responses obtained from one- and three-dimensional models are in good agreement, while differences are observed for the horizontal bending and twisting responses. This may be due to inadequate modelling of the highly non-prismatic hull girder, discontinuities between open and closed cross-section segments, and warping effects [6].

The above state-of-the-art gave incentive to further investigation of this challenging problem. The present paper is intended to propose the consistent theory for hydroelastic coupling between the structural model, represented by either 1D beam model or 3D FEM model, and 3D hydrodynamic model. The well known principles of the modal superposition method are employed. However, the proposed model is slightly different: namely in the way in which hydrostatics is taken into account and in the way the coupling with 3D hydrodynamic model is performed.

The methodology of hydroelastic analysis, which includes definition of the structural model, ship and cargo mass distributions, and geometrical model of ship surface, is shown in Fig.1 according to [7]. First, dry natural vibrations are calculated, and then modal hydrostatic stiffness, modal added mass, damping and modal wave load are determined. Finally, wet natural vibrations are obtained as well as the transfer functions (RAO-response amplitude operator) for determining ship structural response to wave excitation.
2. STRUCTURAL MODEL

The hydroelastic problem can be solved at different levels of complexity and accuracy. The best, but highly time-consuming way, is to consider 3D FEM structural model and 3D hydrodynamic model based on the radiation-diffraction theory. Such an approach is recommended only for the final strength analysis of ship structures. However, in the preliminary strength analysis, it is more rational and convenient to couple 1D FEM model of ship hull with 3D hydrodynamic model.

1D FEM procedure for vertical ship hull vibrations is well known in literature [1]. Coupled horizontal and torsional vibrations are a more complex problem. The matrix finite element equation for coupled natural vibrations yields [8]

\[
\{f\}^{e} = [k]^{e} \{\delta\}^{e} + [m]^{e} \{\delta\}^{e} ,
\]

where \(\{f\}^{e}\) – nodal forces vector, \(\{\delta\}^{e}\) – nodal displacements vector, \([k]^{e}\) – stiffness matrix, \([m]^{e}\) – mass matrix.

These quantities consist of flexural and torsional parts

\[
\{f\}^{e} = \{P\}^{e} , \quad \{\delta\}^{e} = \begin{bmatrix} U \\ V \end{bmatrix}
\]

\[
[k]^{e} = \begin{bmatrix} k_{ss} & 0 \\ 0 & k_{tt} \end{bmatrix} , \quad [m]^{e} = \begin{bmatrix} m_o & m_t \\ m_t & m_o \end{bmatrix}.
\]

Vectors of nodal forces and displacements are

\[
\{P\} = \begin{bmatrix} -Q(0) \\ M(0) \\ Q(t) \\ -M(t) \end{bmatrix} , \quad \{R\} = \begin{bmatrix} -T(0) \\ B_{s}(0) \\ T(t) \\ B_{t}(t) \end{bmatrix}.
\]

In the above formulae symbols \(Q, M, T\) and \(B_{s}\) denote shear force, bending moment, torque and warping bimoment, respectively. Also, \(w, \varphi, \psi\) and \(\theta\) are deflection, rotation of cross-section, twist angle and its variation, respectively. The submatrices have the following meaning:

\[
[k_{ss}] \quad \text{– bending - shear stiffness matrix}
\]

\[
[k_{tt}] \quad \text{– warping - torsion stiffness matrix}
\]

\[
[m_{ss}] \quad \text{– shear - bending mass matrix}
\]

\[
[m_{tt}] \quad \text{– torsion - warping mass matrix}
\]

\[
[m_{st}] = [m_{ts}]^{T} \quad \text{– shear - torsion mass matrix}.
\]

It is evident that coupling between horizontal and torsional vibrations is realised through the mass matrix due to eccentricity of the centre of gravity and shear centre.

Before assembling of finite elements it is necessary to transform Eq. (1) in such a way that all the nodal forces as well as nodal displacement, Eqs. (4) and (5), are related to the first and then to the second node. Furthermore, Eq. (1) has to be transformed from local to global coordinate system. The origin of the former is located at the shear centre, and of the latter at the base line.

Regardless of the FEM approach for the structural model, the governing matrix equation of dry natural hull vibrations yields [9]

\[
([k] - \Omega^2 [m])\{\delta\} = \{0\} ,
\]

where

\[
[k] \quad \text{- stiffness matrix}
\]

\[
[m] \quad \text{- mass matrix}
\]

\[
\Omega \quad \text{- dry natural frequency}
\]

\[
\{\delta\} \quad \text{- dry natural mode}.
\]

As solution of the eigenvalue problem (6), \(\Omega\) and \(\{\delta\}\), are obtained for each \(i\)-th dry mode, where \(i = 1, 2, \ldots, N\), \(N\) is total number of degrees of freedom. Now the natural modes matrix can be constituted

\[
\{\delta\}_i = \{\delta\}_1, \ldots, \{\delta\}_i, \ldots, \{\delta\}_N ,
\]

and the modal stiffness and mass can be determined [9]

\[
[k] = [\delta]^{T} [k] [\delta] , \quad [m] = [\delta]^{T} [m] [\delta] .
\]

Since the dry natural vectors are mutually orthogonal, matrices \([k]\) and \([m]\) are diagonal. Terms \(k_{ii}\) and \(\Omega_{i}^{2}m_{i}\) represent deformation energy and kinetic energy of the \(i\)-th mode, respectively.

Note that generally the first six natural frequencies \(\Omega\) are zero with corresponding eigenvectors representing the rigid body modes. As a result, the first six diagonal elements of \([k]\) are also zero, while the first three elements in \([m]\) are equal to structure mass, the same in all directions \(x, y, z\), and the next three elements represent the mass moment of inertia around the coordinate axes.
3. GEOMETRICAL MODEL OF WETTED SURFACE
For determining pressure forces acting on the wetted surface it is necessary to specify panels and their position in space. Wet surface is given by offsets of waterline ordinates at body plan stations. If the strip method is used for pressure calculation, then the panels bounded with two close stations can be used.

If 3D radiation-diffraction theory is used for hydrodynamic pressure determination, the wetted surface mesh can be created in such a way that panels of more regular shape and refined subdivision in the area of the free surface are achieved [10]. Such a rational mesh is shown in Fig. 2, where the panel rows follow the ship body diagonals similarly to the structural elements of ship outer shell. In this way, the efficiency of hydrodynamic calculation is increased.

**FIGURE 2. RATIONAL MESH OF WETTED SURFACE**

4. DRY MODES OF WETTED SURFACE
As mentioned in Section 2, structural dry modes can be determined by 1D or 3D FEM analysis. If 1D analysis is used, the beam modes are spread to the ship wetted surface as follows.

Vertical vibration
\[ h_y = -\frac{d}{dx}(Z - z_N) + w_y k, \]
\[ (9) \]

horizontal vibration
\[ h_h = \frac{d}{dx}w_y i + w_h j, \]
\[ (10) \]

torsional vibration
\[ h_t = \psi(Z - z_S) j - \psi Y k, \]
\[ (11) \]

where \( w \) is hull deflection, \( \psi \) is twist angle, \( Y \) and \( Z \) are coordinates of the point on ship surface, and \( z_N \) and \( z_S \) are coordinates of neutral line and shear centre respectively.

If strong coupling between horizontal and torsional vibration occurs, as in the case of container ships, the coupled mode yields
\[ h_{ht} = \left( -\frac{d}{dx}w_y + \frac{d}{dx} \psi \right) i + \frac{d}{dx}w_h + \psi(Z - z_S) j - \psi Y k, \]
\[ (12) \]

where \( \psi = \psi(x, y, Z) \) is the cross-section warping function reduced to the wetted surface [11].

5. HYDRODYNAMIC MODEL
5.1. General
The fully consistent and efficient seakeeping model with forward speed, even for rigid body, does not exist yet and only the approximate models are employed for the time being. These approximated models spread from the so called 2D strip theories, over so called encounter frequency approximation (main effect is put on the change of frequency due to speed) and different 3D approximations based on the exact forward speed Green function with or without coupling with the ship steady flow, to non-linear time domain seakeeping models. However, according to the authors’ knowledge, none of the above methods has been validated enough, and the problem is still open.

The most widely used method, the one based on the encounter frequency approximation, is applied in this study, even if the method utilizing the exact forward speed Green function (together with so called Neuman-Kelvin approximation) is also implemented in the software used [12].

The choice of hydrodynamic model is not likely to change the general conclusions of this paper, because the experience shows that, in the case of a rigid body, the encounter frequency method agrees quite well with more complicated methods, at least for the global ship loadings. This, however, does not mean that we should forget this important assumption, and that point should be kept in mind when more detailed correlations with experiments and full scale results will be undertaken.

Anyhow, the coupling procedure does not depend on the used hydrodynamic model, and is therefore described here for the zero speed case, as the simplest one.

5.2. Theoretical Background
The harmonic hydroelastic problem is considered in frequency domain and therefore we operate with amplitudes of forces and displacement. In order to perform coupling of the structural and hydrodynamic models, it is necessary to subdivide the external pressure forces in a convenient manner [13]. First, the total hydrodynamic force \( F^h \) has to be split into two parts: the first part \( F^R \) depending on the structural deformations, and the second one \( F^{DI} \) representing pure excitation
\[ F^h = F^R + F^{DI}. \]
\[ (13) \]

Furthermore, the modal superposition method can be used. The vector of the wetted surface deformations \( \mathbf{H}(x, y, z) \) can be presented as a series of dry natural modes \( \mathbf{h}_i(x, y, z) \)
\[ \mathbf{H}(x, y, z) = \sum_{i=1}^{N} \xi_i \mathbf{h}_i(x, y, z) \]
\[ (14) \]

where \( \xi_i \) are unknown modal coefficients. Vectors \( \mathbf{h}_i(x, y, z) \) related to wetted surface are obtained from the structural dry modes as explained in the previous section.

As far as the hydrodynamic part of the hydroelastic problem is concerned, the potential flow theory is adopted. Within this approach the fluid is assumed inviscid and fluid flow irrotational, so that the velocity potential can be defined and the corresponding boundary
value problem can be formulated. The general seakeeping problem for ship advancing with forward speed in waves is an extremely difficult problem and only approximate solutions exist today. In this paper we do not enter into the details of the pure hydrodynamic analysis and we concentrate on the coupling principles only. Indeed, the coupling procedure remains the same regardless of the complexity of the boundary value problems for velocity potentials.

Thus, this coupling procedure is illustrated for the seakeeping problem without forward speed. The total velocity potential \( \phi \), is defined with the Laplace differential equation and the given boundary values

\[
\Delta \phi = 0 \quad \text{within the fluid} \\
-\nu \phi + \frac{\partial \phi}{\partial z} = 0 \quad \text{at the free surface, } z = 0 \\
\frac{\partial \phi}{\partial n} = -i \omega H n \quad \text{on the wetted body surface, } S,
\]

where \( \nu \) is the wave number, \( \nu = \omega / g \), \( \omega \) is wave frequency, \( n \) is the wetted surface normal vector, and \( i \) is the imaginary unit.

Furthermore, the linear wave theory enables the following decomposition of the total potential

\[
\phi = \phi_I + \phi_D - i \omega \sum_{j=1}^{N} \xi_j \phi_{Rj},
\]

where

\[
\phi_I = -i \frac{gA}{\omega} e^{i \nu(z + ix)} - \text{incident wave potential} \\
\phi_D = -i \text{diffraction wave potential} \\
\phi_{Rj} = -i \text{radiation wave potential} \\
A = \text{wave amplitude}
\]

Now, the body boundary conditions (15) can be deduced for each potential

\[
\frac{\partial \phi_D}{\partial n} = \frac{\partial \phi_I}{\partial n} = \frac{\partial \phi_{Rj}}{\partial n} = h n.
\]

It is necessary to point out that the diffraction and radiation potentials should also satisfy the radiation condition at infinity.

Once the potentials are determined, the modal hydrodynamic forces are calculated by pressure work integration over the wetted surface. The total linearised pressure can be found from Bernoulli’s equation

\[
p = i \omega \phi_c - \rho g z.
\]

First, the term associated with the potential \( \phi \) is considered and subdivided into excitation and radiation parts (16)

\[
F_i^{DI} = i \omega \int_S \phi_I n d S,
\]

\[
F_i^R = \rho \omega^2 \sum_{j=1}^{N} \xi_j A_{jj} ,
\]

Thus, (20) represents the modal pressure excitation. Now one can decompose (21) into the modal inertia force and the damping force associated with acceleration and velocity respectively

\[
F_i^a = \text{Re}(F_i^R) = \omega^2 \sum_{j=1}^{N} \xi_j A_{jj},
\]

\[
F_i^v = \text{Im}(F_i^R) = \omega \sum_{j=1}^{N} \xi_j B_{jj}.
\]

where \( A_{jj} \) and \( B_{jj} \) are elements of added mass and damping matrices, respectively.

Determination of added mass and damping for rigid body modes is a well-known procedure in ship hydrodynamics [1]. Now the same procedure is extended to the calculation of these quantities for elastic modes.

The hydrostatic part of the total pressure, \( -\rho g z \) in (19) is considered within the hydrostatic model.

### 6. HYDROSTATIC MODEL

#### 6.1. Pressure Forces

Hydroelastic analysis is performed by the modal superposition method. Modal forces represent work of actual static and dynamic forces on rigid body and elastic mode displacements. Thus, modal restoring forces consist of time-dependent modal pressure forces and modal gravity forces. In order to specify the contribution of the hydrostatic part of the total pressure, \( -\rho g z \) in (19), to hydrostatic stiffness it is necessary to determine the change of the modal hydrostatic force as the difference between its instantaneous value and the initial value for the vibration mode \( h \), of the body wetted surface \( Z = Z(x, y) \)

\[
F_i^H = -\rho g \int_S \left( Z h n + Z \delta h n + Z h \delta n \right) d S.
\]

Each of the above quantities can be presented in the form, \( (\ldots) = (\ldots) + \delta(\ldots) \) where \( \delta \) denotes the variation. By neglecting small terms of higher order, one can write for the modal hydrostatic force (24)

\[
F_i^H = -\rho g \int_S (\delta Z h n + Z \delta h n + Z h \delta n) d S.
\]

Variation of the particular quantity can be determined by applying the notion of directional derivative \( \hat{H} \nabla \), where \( H \) is given with (14) and \( \nabla \) is Hamilton differential operator

\[
\nabla = \sum_{j=1}^{N} \xi_j \int h^j x \frac{\partial}{\partial x} + h^j y \frac{\partial}{\partial y} + h^j z \frac{\partial}{\partial z}.
\]
As a result
\[
\delta Z = (HV)Z = Hk, \quad \delta n = (HV)n.
\] (27)

Determining the variation of the body surface normal vector, \(\delta n\), according to (27) is a rather difficult task. Therefore, a relatively simpler procedure, taken from [14], is applied. By using Eqs. (26), (27) and \(\delta n\) from [14], the modal hydrostatic force (25) can be presented in the following form:
\[
\delta F_i^m = - \sum_{j=1}^{N} \xi_j C_{ij} \delta n = - \sum_{j=1}^{N} \xi_j C_{ij} \delta n.
\] (28)

where
\[
C_{ij} = C_{ij}^H + C_{ij}^H + C_{ij}^H
\] (29)
is the \(i,j\)-th element of the hydrostatic stiffness matrix, composed of static pressure, surface mode and normal vector contributions, respectively
\[
C_{ij}^H = \rho \|Z\| \int_{S} \left( \frac{\partial h_i^j}{\partial x} + \frac{\partial h_i^j}{\partial y} + \frac{\partial h_i^j}{\partial z} \right) n \times
\] (30)

\[
C_{ij}^H = \rho \|Z\| \int_{S} \left( \frac{\partial h_i^j}{\partial x} + \frac{\partial h_i^j}{\partial y} + \frac{\partial h_i^j}{\partial z} \right) n \times
\] (31)

\[
C_{ij}^H = \rho \|Z\| \int_{S} \left( \frac{\partial h_i^j}{\partial x} + \frac{\partial h_i^j}{\partial y} + \frac{\partial h_i^j}{\partial z} \right) n \times
\] (32)

Note that the wetted surface coordinate \(Z\) is measured from the waterplane. Based on the constitution of the above coefficients, it is evident that the hydrostatic stiffness matrix is not diagonal.

In literature, various definitions of hydrostatic stiffness matrix can be found [15], [16]. The advantage of the present method is that the derived formulae are general and applicable for a complex body, as well as for its parts. This is important for determining the local hydrostatic action as internal loads, transfer of load to a FEM structural model, etc.

6.2. Gravity forces
The above expressions represent only the action of the hydrostatic pressure, and the gravity part has to be added in order to complete the total restoring coefficients. Similarly to the pressure part, Eqs. (24) and (25), a change of the generalised modal gravity force associated with a particular mode yields [14]
\[
F_{i}^m = -g \int_{V} (HV) h_i^j dV = - \sum_{j=1}^{N} \xi_j C_{ij}^m
\] (33)

where \(V\) is body volume and \(dV\) is differential mass.

By employing (27) and further (26) one can write
\[
F_{i}^m = -g \int_{V} (HV) h_i^j dV = - \sum_{j=1}^{N} \xi_j C_{ij}^m
\] (34)

where
\[
C_{ij}^m = g \int_{V} \left( \frac{\partial h_i^j}{\partial x} + \frac{\partial h_i^j}{\partial y} + \frac{\partial h_i^j}{\partial z} \right) n \times
\] (35)

6.3. Restoring Stiffness
Finally, the complete restoring coefficients read
\[
C_{ij} = C_{ij}^H + C_{ij}^m
\] (36)

It is important to point out that the above expressions for the hydrostatic and gravity coefficients are general and therefore valid not only for the elastic modes but also for the rigid body modes as well as for their coupling [14].

7. HYDROELASTIC MODEL
After the structural, hydrostatic and hydrodynamic models have been determined, the hydroelastic model can be constituted. For that purpose, let us impose modal hydrodynamic forces (21), (22) and (23) and hydrostatic and gravity forces (28), (33) to the modal structural model, Section 2
\[
(k - \omega^2 [m])[\xi] = [F]^D + [F]^a + [F]^V + [F]^H + [F]^m
\] (37)

Furthermore, all terms dependent on unknown modal amplitudes, \(\xi\), can be separated on the left hand side. Thus, the governing matrix differential equation, which combines rigid body motions and vibrations, is deduced
\[
[k] - i \omega \left[ C - i \omega a [d] + [B(a)] \right] - \omega^2 [m] [A(a)] [\xi] = [F].
\] (38)

where all quantities are related to the dry modes:

- \([k]\) - structural stiffness
- \([d]\) - structural damping
- \([m]\) - structural mass
- \([C]\) - restoring stiffness
- \([B(a)]\) - hydrodynamic damping
- \([A(a)]\) - added mass
- \([\xi]\) - modal amplitudes
- \([F]\) - wave excitation
- \(\omega\) - encounter frequency.
Structural damping can be given as a percentage of the critical value based on experience with similar ships. As it is well-known, added mass and hydrodynamic damping depends on the frequency. The solution of (38) gives the modal amplitudes $\xi_i$ and displacement of any point of the structure obtained by retracking to (14).

The wet natural modes can also be determined by solving the eigenvalue problem extracted from (38)

$$\left\{ [k] + [C] - \omega^2 ([m] + [A(\omega)]) \right\} \{ \xi \} = [0].$$

(39)

Now damping is neglected since its influence on the eigenpair is very small. The solution of (39) gives natural frequencies of ship motion and vibration in water and the corresponding so-called wet natural modes. Since added mass is a frequency dependent function, it is evident that an iteration procedure has to be employed to solve (39). Therefore, the wet modes are not orthogonal. Also, there are no more zero natural frequencies and pure rigid body modes due to their coupling with the elastic modes [13].

8. HYDROELASTICITY OF A FLEXIBLE BARGE

The implementation of the structural model in hydroelastic analysis is checked in case of a flexible barge, for which test results are available [7], [13]. The barge consists of 12 pontoons, which are connected by a steel rod above the deck level, Figure 3.

The deformation centre is above the gravity centre and strong coupling between horizontal and torsional vibrations is achieved as in case of container ships. For illustration, Figures 4 and 5 show correlation of the calculated and the measured transfer function of horizontal bending moment and torque, respectively, as function of wave period $T$, [13], [17].

9. HYDROELASTICITY ANALYSIS OF CONTAINER SHIP

9.1. Ship Particulars

The application of developed theory is illustrated in case of a 7800 TEU VLCS (Very Large Container Ship), Figure 7. The main vessel particulars are the following:

- Length overall $L_{ovl} = 334$ m
- Length between perpendiculars $L_{pp} = 319$ m
- Breadth $B = 42.8$ m
- Depth $H = 24.6$ m
- Draught $T = 14.5$ m
- Displacement, full load $\Delta_p = 135336$ t
- Displacement, ballast $\Delta_b = 68387$ t
- Engine power $P = 69620$ kW
- Ship speed $v = 25.4$ kn
FIGURE 7. 7800 TEU CONTAINER SHIP

The ship hull stiffness properties are calculated by program STIFF, based on the theory of thin-walled girders [11]. The geometrical properties rapidly change values in the engine and superstructure area due to closed ship cross-section. This is especially pronounced in case of torsional modulus, which takes quite small values for open cross-section and rather high for the closed one [18].

9.2. Natural Vibrations

Dry natural vibrations are calculated by program DYANA [18]. The ship hull is divided into 50 beam finite elements. Finite elements of closed cross-section (6 d.o.f.) are used in the ship bow, ship aft and in the engine room area. Dry natural frequencies for vertical vibrations, and coupled horizontal and torsional vibrations for full load and ballast conditions are listed in Table 1. Their values for ballast condition are higher due to the lower mass. The lowest frequency value, which plays the main role in wave excitation, is detected for coupled vibrations. As a result of rather low torsional stiffness, this value belongs to primarily torsional mode.

TABLE 1. DRY NATURAL FREQUENCIES, $\omega_i$ [rad/s]

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>Full load</th>
<th>Ballast</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertical</td>
<td>Horizontal + torsional</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>2.18</td>
</tr>
<tr>
<td>2</td>
<td>8.41</td>
<td>4.23</td>
</tr>
<tr>
<td>3</td>
<td>13.22</td>
<td>7.08</td>
</tr>
<tr>
<td>4</td>
<td>18.07</td>
<td>9.23</td>
</tr>
<tr>
<td>5</td>
<td>23.04</td>
<td>13.19</td>
</tr>
<tr>
<td>6</td>
<td>28.09</td>
<td>15.37</td>
</tr>
<tr>
<td>7</td>
<td>32.77</td>
<td>18.22</td>
</tr>
<tr>
<td>8</td>
<td>37.22</td>
<td>22.65</td>
</tr>
<tr>
<td>9</td>
<td>41.73</td>
<td>23.75</td>
</tr>
<tr>
<td>10</td>
<td>42.27</td>
<td>28.38</td>
</tr>
</tbody>
</table>

Once the dry natural modes of ship hull are determined it is possible to transfer the beam node displacements to the ship wetted surface for the hydrodynamic calculation, Eqs. (9) and (12). The first two dry natural modes of the ship wetted surface in case of vertical and coupled horizontal and torsional vibrations for full load are shown in Figures 8, 9, 10 and 11.

In addition, the values of natural frequencies of wet ship in full load condition are listed in Table 2. By comparing them to those of dry ship in Table 1, one sees that vertical vibration frequencies are more reduced than those of coupled horizontal and torsional vibrations.

TABLE 2. WET NATURAL FREQUENCIES, $\omega_i$ [rad/s]

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>Full load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertical</td>
</tr>
<tr>
<td>1</td>
<td>3.07</td>
</tr>
<tr>
<td>2</td>
<td>6.33</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>13.96</td>
</tr>
<tr>
<td>5</td>
<td>18.09</td>
</tr>
<tr>
<td>6</td>
<td>22.18</td>
</tr>
<tr>
<td>7</td>
<td>26.23</td>
</tr>
<tr>
<td>8</td>
<td>30.48</td>
</tr>
<tr>
<td>9</td>
<td>35.01</td>
</tr>
<tr>
<td>10</td>
<td>41.23</td>
</tr>
</tbody>
</table>

FIGURE 8. THE FIRST NATURAL MODE OF VERTICAL VIBRATIONS, $\omega_1$=4 rad/s

FIGURE 9. THE SECOND NATURAL MODE OF VERTICAL VIBRATIONS, $\omega_2$=8.41 rad/s

FIGURE 10. THE FIRST NATURAL MODE OF COUPLED HORIZONTAL AND TORSIONAL VIBRATIONS, $\omega_1$= 2.18 rad/s
9.3. Ship Response

Numerical calculation of ship response to waves is performed for full load and ballast condition, unit harmonic wave amplitude, and set of heading angles, ship speed and wave length [18]. Here, only some selected results for full load and ship speed of $U = 25$ kn are presented. Vertical response is shown for head sea, $\gamma = 180^\circ$, and coupled horizontal and torsional response for quartering sea, $\gamma = 120^\circ$. In all figures hydroelastic response is correlated to rigid body response determined by program HYDROSTAR [12], where the so called encounter frequency approximation was used for the solution of the seakeeping problem with forward speed. Due to the lack of measured data on large container vessels, modal damping that is usually used in global vibration calculations of merchant ships, i.e. 2% of the critical value, is applied in the analysis [19]. Figure 12 shows the transfer function of the vertical bending moment at the midship section. Heave resonance occurs at encounter frequency $\omega_e = 0.83$ rad/s, while bending resonance is achieved at $\omega_e = 3.74$ rad/s. In the former case, bending moments determined by hydroelasticity and rigid body motion are almost the same, Figure 13. In the latter case, the hydroelastic bending moment takes very high values compared to the rigid body one, Figure 14.

The results of coupled horizontal and torsional ship response are presented in Figures 15 to 20. Figures 15 and 16 show transfer functions of horizontal bending moment and torque referred to the shear (torsional) centre, at the midship section, respectively. Hydroelastic effect is more pronounced in torsional than in bending response.
Large relative discrepancies between the hydroelastic and rigid body torque in low frequency domain in Figure 16 might be the result of ill-conditioning of the former. In hydro-rigid-body analysis, namely, sectional forces are determined by pressure integration over wetted surface, while in hydroelastic analysis sectional forces are obtained by derivation of hull displacements, which are predominantly rigid body ones in low frequency domain. Looking through the modal decomposition, derivatives of rigid body modes are zero, while those of elastic body are quite small with decreased accuracy.

Distributions of bending moment in case of rigid body and elastic resonance are shown in Figures 17 and 18. In the former case hydroelastic and rigid body bending moments are quite close, while in the latter case the elastic bending moment is much higher than the rigid body one. Similar situation occurs with torques, Figures 19 and 20. When elastic response is in resonance with wave excitation the hydroelastic torque takes very high values.

**9.4. Validation Of 1D FEM Model**

For this purpose, the light weight loading condition of dry ship with displacement $\Delta = 33692$ t is considered. The first dominantly torsional mode and the second dominantly horizontal flexural mode of the wetted surface are quite similar to those shown in Figures 10 and 11, obtained for the full load. The reliability of 1D FEM analysis is verified by 3D FEM analysis of the considered ship [17]. The first two 3D dry natural modes are similar to those of 1D analysis, Figures 10 and 11. The corresponding natural frequencies obtained by 1D and 3D analyses are compared in Table 3. Quite good agreement is achieved. Values of natural frequencies for higher modes are difficult to correlate, since strong coupling occurs between global hull modes and local substructure modes of 3D analysis.

**TABLE 3. NATURAL FREQUENCIES OF CONTAINER SHIP IN LIGHT WEIGHT CONDITION, $\omega$ [rad/s]**

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>1 (T+HB)</th>
<th>2(HB+T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D FEM</td>
<td>5.39</td>
<td>9.23</td>
</tr>
<tr>
<td>3D FEM</td>
<td>5.41</td>
<td>9.42</td>
</tr>
</tbody>
</table>
10. CONCLUSION

Ultra large container ships are quite flexible and stretch the bounds of present classification rules for reliable structure design. Therefore, hydroelastic analysis has to be performed. One of the basic steps is dry natural vibration analysis of ship hull. Vertical vibration calculation is performed in a standard way, while the coupled horizontal and torsional vibration is rather complex.

The performed vibration analysis of the 7800 TEU Container ship shows that the developed hydroelasticity theory, utilizing 1D FEM structural model and 3D hydrodynamic model, is an efficient tool for application in ship hydroelastic analyses. The obtained results point out that the transfer functions of hull sectional forces in case of resonant vibration (springing) are much higher than in resonant ship motion. This is the main issue of the paper, which requires further investigation.

In order to complete hydroelastic analysis of container ships and confirm its importance for ship safety, it is necessary to proceed further to ship motion calculation in irregular waves for different sea states, based on the known transfer functions. This includes determination of global wave loads, i.e. bending and torsional moments and their conversion into stresses, stress concentration in critical areas of ship structures, especially in hatch corners due to suspended warping, and fatigue of structural details. The same numerical procedures, which are used in hydro-rigid-body analysis, are at disposal and are applicable in this case.

The used beam model of ship hull is a reasonable choice for determining wave load. However, stress concentration in hatch corners calculated directly by the beam model is underestimated [20,21]. This problem can be overcome by applying substructure approach, i.e. 3D FEM model of substructure with imposed boundary conditions from beam response. In any case, 3D FEM model of complete ship is preferable from the viewpoint of determining stress concentration.

Another aspect of hydroelasticity is slamming and whipping, which are related to vertical bending. This impulsive load is considered in time domain and is outside the scope of this paper. However, it must be considered within general container ship problematic.

At the end of a complete investigation, which also has to include model tests and full scale measurements, it will be possible to decide on the extent of the revision of Classification Rules for the design and construction of ultra large container ships. Also, operational conditions for this type of ships, i.e. reduction of ship speed and change of heading angle for some sea states due to resonant flexural and torsional response (as well as parametric rolling) have to be reconsidered.

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REFERENCES


