An efficient hydroelastic model for wave induced coupled torsional and horizontal ship vibrations
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Introduction

The wave induced hydroelastic response of a ship becomes an important issue for large ships, because of the structural natural frequencies which can fall into the range of the frequencies contained in the typical sea spectra. In those cases it is likely that the ship will experience an important hydroelastic response, usually called springing. For those ships with closed cross-section (tankers, bulk, general cargo, ...) the lowest natural frequencies are usually associated with the vertical bending, while for the ships with open cross-section, such as container ships, the lowest elastic natural frequency might be the coupled mode of torsion and horizontal bending. Also, for this type of ships coupling between horizontal and torsional vibration is very pronounced due to the fact that center of torsion (shear center) is below the keel, Figure 1. Since only the first few elastic modes (with lowest natural frequencies) may be excited, the simple non-uniform beam model of the ships is usually considered to be sufficient for springing analysis. However, the beam model is sufficiently simple only in cases of uncoupled responses so that for the container ships the full 3D FEM models are usually preferred, Figure 2. Since the 3D FEM modeling is very complex and time consuming, there is a need for simplified structural models which can be used in the pre-design process. Here we propose an improved beam model for coupled torsional and horizontal vibrations, and we use it together with 3D hydrodynamic panel code HYDROSTAR.

Figure 1: Initial and deformed ship section.

Mathematical model

Structural model

The basic definitions are shown in Figure 1. The $w$ denotes the horizontal deflection, $\psi$ the twist angle, $G_0$ and $G$ the initial and final position of the center of gravity and $C_0$ and $C$ the initial and final position of the center of torsional rotation (shear center). The coupled system of differential equations for prismatic beam becomes [4]:

$$EI \frac{\partial^4 w}{\partial x^4} - \left( \frac{EI}{GF} m + J \right) \frac{\partial^4 w}{\partial x^2 \partial t^2} + m \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2 \partial t^2} \right) + \frac{J}{GF} \frac{\partial^4 w}{\partial t^4} + \frac{z_{CG}}{GF} \frac{\partial^2 \psi}{\partial x^2} = q$$  (1)

$$EI \frac{\partial^4 \psi}{\partial x^4} - GJ \frac{\partial^2 \psi}{\partial x^2} + J_t \frac{\partial^2 \psi}{\partial t^2} - J_w \frac{\partial^4 \psi}{\partial x^2 \partial t^2} + m z_{CG} \left( \frac{\partial^2 w}{\partial t^2} - \frac{EI}{GF} \frac{\partial^4 w}{\partial x^2 \partial t^2} \right) = \mu$$  (2)
where $E$ and $G$ are Young’s and shear modulus of elasticity, $I$ and $F$ are bending modulus and shear area, $m$ is the distributed ship mass per unit length, $J$ is mass moment of inertia about $z$ axis, $I_w$ is warping modulus, $I_t$ is torsional modulus, $J_t$ is polar mass moment of inertia, $J_{tw}$ is mass bimoment of inertia, $q$ is the distributed lateral load and $\mu$ is the distributed torsional moment.

Harmonic excitation is assumed and the finite element method is used for solving the above coupled differential equations. The resulting matrix equation takes the following form:

$$\begin{bmatrix} K \ - \ \omega^2 M \end{bmatrix} \{U\} = \{P\}$$  \hspace{1cm} (3)

where $[M]$ and $[K]$ are the mass and stiffness matrix respectively, $\omega$ is the excitation frequency, $\{U\}$ is the displacement vector containing the amplitudes of both horizontal deflections and torsional rotations and their first derivatives, and $\{P\}$ is the amplitude vector of the external hydrodynamic pressure loads.

The corresponding eigenvalue problem is then formulated:

$$\begin{bmatrix} K \ - \ \omega^2 M \end{bmatrix} \{u\} = \{0\}$$  \hspace{1cm} (4)

and the, so called, ”dry” natural frequencies and modes are defined:

$$\omega_i, \ \{u\}_i, \ i = 1, N$$  \hspace{1cm} (5)

$$[\Xi] = [\{u\}_1, \{u\}_2, ..., \{u\}_i, ..., \{u\}_N]$$  \hspace{1cm} (6)

The forced vibration problem may be solved by the mode superposition method. In that case, the displacement vector is expressed as series of the natural modes:

$$\{U\} = [\Xi] [\xi]$$  \hspace{1cm} (7)

where $[\xi]$ is the vector of modal amplitudes.

By multiplying (4) with $[\Xi]^T$ from the left, and substituting (7) the modal matrix equation for forced ship vibration becomes:

$$\begin{bmatrix} k \ - \ \omega^2 m \end{bmatrix} [\xi] = \{F^h\}$$  \hspace{1cm} (8)

where $[k]$ and $[m]$ are the modal stiffness and mass matrix respectively and $\{F^h\}$ is the modal loads:

$$[m] = [\Xi]^T [M] [\Xi], \ [k] = [\Xi]^T [K] [\Xi], \ \{F^h\} = [\Xi]^T \{P\}$$  \hspace{1cm} (9)

Note that, as result of orthogonality, the matrices $[k]$ and $[m]$ are diagonal.

**Hydrodynamic model and coupling**

The above mentioned modal superposition method is used for coupling of structural and hydrodynamic models. In that respect the displacement of any point belonging to the ship structure is found by the following expression:

$$H(x, y, z) = \sum_{i=1}^{N} \xi_i h_i^x(x, y, z) + \sum_{i=1}^{N} \xi_i h_i^y(x, y, z) j + h_i^z(x, y, z) k$$  \hspace{1cm} (10)
where the vector functions $h^i$ denotes the dry structural modes and $\xi_i$ their amplitudes. The functions $h^i$ transfer the modal beam displacement vector $\{u\}_i$ from the beam center of deformation (point C in Figure 1) to the ship cross-section and subsequently to whole ship structure.

We adopt the potential theory and define the velocity potential $\varphi(x)$ by the following boundary value problem (BVP):

$$\begin{align*}
\Delta \varphi &= 0 \quad \text{in the fluid} \\
-\nu \varphi + \frac{\partial \varphi}{\partial z} &= 0 \quad z = 0 \\
\frac{\partial \varphi}{\partial n} &= -i\omega H n \quad \text{on } S_B
\end{align*}$$

where $\nu$ is the wavenumber $\nu = \omega^2/g$.

Thanks to the linearity, we decompose the total potential as follows:

$$\varphi = \varphi_I + \varphi_D - i\omega \sum_{j=1}^N \xi_j \varphi_{Rj}$$

From (10) and (11), we can deduce the body boundary conditions for each potential:

$$\frac{\partial \varphi_D}{\partial n} = -\frac{\partial \varphi_I}{\partial n}, \quad \frac{\partial \varphi_{Rj}}{\partial n} = h_j n$$

Let us finally note, that the diffraction and radiation potentials should also satisfy the radiation condition. The classical 3D panel method, based on source formulation, is used to solve the above defined BVP’s using the numerical code HYDROSTAR. Once the different potentials are found, the pressure is calculated from Bernoulli’s equation:

$$p = i\omega \rho \varphi - \rho g z$$

and the corresponding forces are obtained after integration over the wetted body surface. The forces are then subdivided into the part independent of ship motions/deformations and the parts depending on the ship motions/deformations. We write (see [2] for details):

$$\{F^h\} = \{F^{D_I}\} + (\omega^2 [A] + i\omega [B] - \omega [C])\{\xi\}$$

The coupled motion equation can now be written in the following form:

$$\{-\omega^2([m] + [A]) - i\omega [B] + ([k] + [C])\} \{\xi\} = \{F^{D_I}\}$$

where:

- $[m]$ - modal genuine mass
- $[k]$ - modal structural stiffness
- $[A]$ - hydrodynamic added mass
- $[B]$ - hydrodynamic damping
- $[C]$ - hydrostatic stiffness
- $\{\xi\}$ - modal amplitudes
- $\{F^{D_I}\}$ - modal pressure excitation

The solution of the above equation gives the motion amplitudes $\xi_i$ and the problem is formally solved.

**Preliminary results and comments**

Here below we show some preliminary results for simplified barge with open section through the midship part but closed at its ends. The geometry of the barge is presented in Figure 3 (the wall thickness is 1cm), together with its first three dry natural frequencies. Relatively good agreement between the 1D
Figure 3: Barge geometry and the first 3 dry natural frequencies [Hz].

and 3D results is found. The corresponding mode shapes, obtained with 1D beam model are shown in Figure 4. All the modes show an important coupling between the torsion and horizontal bending. The first and second modes are dominated by the torsion, while the third one is dominated by the horizontal bending.

![Figure 4: Natural modes.](image)

**References**


