ABSTRACT
Weakly nonlinear numerical model, able to predict the parametric roll behavior of ships, is presented and validated by comparisons with experiments. The numerical model is based on the transfer of frequency domain data to time domain (e.g. see Malenica et al. (2003.) using the procedure proposed by Cummins (1962.), and on the inclusion of so called Froude Krylov nonlinear part of the loading in time domain. The experiments were conducted in MARIN Research Institute, where the 300m container vessel was tested. The comparisons of the numerical calculations and experimental results clearly shows the ability of this, relatively simple model, to predict this kind of nonlinear ship behavior.

1 INTRODUCTION
The parametric roll of ships is well identified problem, and lot of litterature exists on the subject. The problem was reactualized after the spectacular accident which happened on APL China Container vessel in October 1998. This ship encountered unexpected roll of about 40 degrees and lost about one third of the deck containers. After careful investigations, which involved both numerical simulations and experiments, it was found that parametric roll was the likely cause of damage. It should also be noted that several other accidents on different ship types, not reported in the literature, are also attributed to this phenomena.

Figure 1: Observed damages after extreme roll motion of a container vessel.

Parametric roll usually occurs in head or following waves and due to this, it can not be explained by linear mechanism because, in these conditions (nearly head or following waves), the linear excitation is very low (zero for purely head or following seas). The source of excitation is found in the temporal changes of the ship stability characteristics when the waves are passing the ship.
The simplest mathematical model of parametric roll considers the single degree of freedom motion equation in which the restoring is made nonlinear.

\[ \ddot{\phi} + 2\mu \dot{\phi} + \omega_0^2 [1 + f(t)] \phi = 0 \]  

(1)

where \( \phi \) denotes the roll angle, \( \mu \) is the damping coefficient, \( \omega_0 \) is the roll natural frequency and \( f(t) \) denotes the nonlinear restoring coefficient.

The definition and calculation of the nonlinear restoring coefficient is not unique and several approximations are possible. The simplest assumption is that the nonlinear restoring is the harmonic function of time with certain amplitude. This assumption is supported by the usual shape of the curve representing the variation of metacentric height (\( GM \)) with respect to the instantaneous (static) wave position. We write:

\( f(t) = \frac{\delta GM}{GM_0} \cos(\omega t + \epsilon) \)  

(2)

One typical example of \( GM \) variation in waves is shown in Figure 2. Note that the determination of this curve is not very simple and usually requires the remeshing of the hydrodynamic model at each time step i.e. for each wave position. It should be clearly noted that this is just one approximation of the real situation because the \( GM \) variation depends also on other quantities (heave motion, pitch motion, instantaneous roll angle, instantaneous wave elevation, ...) and those dependencies are all nonlinear. However, the above analysis allows us to get simple understanding of what the parametric roll is. Indeed, after introducing the equation (2) in (1), we end up with the Mathieu type instability equation which is well known in other fields of physics and efficient analysis tools exist. In particular, the Mathieu equation allows for the definition of the stability zones. The exact determination of these zones is not trivial, but the first order approximation gives very simple diagram shown in Figure 3. The abscissa in this graph represents the ratio of the excitation frequency (wave encounter frequency \( \omega_e \)) and the roll natural frequency, while the ordinate represents the amplitude of \( GM \) variation i.e. the amplitude of parametric roll excitation. These are two main parameters for parametric roll analysis, and we can easily deduce the critical conditions which increase the probability for parametric roll to happen:

\[ \omega_e \approx 2\omega_0 \quad , \quad \lambda \approx L \quad , \quad \mu << \]  

(3)

where \( \lambda \) denotes the wave length and \( L \) is the ship length.

When the excitation frequency is close to twice the roll natural frequency the stability region is reduced and even small excitation amplitude may excite the unstable roll motion. On the other hand, the risk of parametric roll increase with increasing amplitude of \( GM \) variation and this variation is usually most important if the wave length is approximately equal to the ship length.
and if the ship is sailing in head or following seas. Finally the role of damping is always beneficial since it reduce the instability zone. In the above procedure, for the sake of simplicity, the roll damping was taken as a linear in order to derive simple expressions for stability zones. Usually the roll damping is nonlinear, at least quadratic, so that the exact shape of the stability zones will change.

Anyway, it is now easier to understand why the parametric roll phenomena particularly affects the container ships. Indeed, the container ships have very low block coefficient which makes the amplitude of $GM$ variation more important so that the ship operating conditions are closer to the instability zone.

2 NUMERICAL MODEL

In the previous section, we presented the simplified analysis of the parametric roll phenomena in order to get the basic insight into the physics of the problem. It is clear that this simplified analysis is very limited since it involves several approximations which were briefly discussed. That is why, in the final phase of parametric roll evaluation, more sophisticated numerical methods should be used since they are able to include all kinds of nonlinearities in a natural way. The price to pay is the complexity of the models and relatively large CPU time. Several more or less sophisticated nonlinear numerical models exist and all of them include at least the recalculation of the complete hydrostatic matrix at each time step. Here below we present one of those methods which is in use in Bureau Veritas.

The overall computational scheme is shown in Figure 4. The calculations start with the linear frequency domain analysis by 3D panel code HYDROSTAR. The result of these calculations are the linear hydrodynamic coefficients: added mass $A(\omega)$, damping $B(\omega)$ and excitation $F_{DI}(\omega)$. From these, frequency dependent coefficients, the transfer to the time domain is performed using the method proposed by Cummins (1962) in the way presented in Malenica et al. (2003.), and the following motion equation is derived:

$$\begin{align*}
(M + A_\infty)\ddot{\xi}(t) + C \dot{\xi}(t) + \int_0^t K(t - \tau) [\dot{\xi}(\tau)] d\tau &= F_{DI}(t) + F_{NL}(t) \\
\end{align*}$$

Figure 3: Stability zones.

where overdots denote the time derivatives, $[M]$ is the ship mass matrix, $[A_\infty]$ is the infinite frequency added mass matrix, $[K(t - \tau)]$ is the matrix of the impulse response functions, $[C]$ is the linear restoring, $F_{DI}$ is the linear part of excitation and $F_{NL}$ denotes the nonlinear forces whatever they are.

All the elements in this equation, except the nonlinear forces, can be calculated using the frequency domain coefficients evaluated by HYDROSTAR. The advantage of the time domain method lies in the possibility of introducing the non-linear components in the excitation forces $F_{NL}(t)$. The motion equation (4) is integrated in time using the Runge Kutta 4th order scheme.

One of critical points in the above scheme is the evaluation of the nonlinear Froude Krylov forces. The Froude Krylov approximation basically means the inclusion of the hydrostatic pressure varia-
tion up to the instantaneous incident wave elevation, instead up to the mean free surface \((z = 0)\) as it is done within the linear model. In this way the \(GM\) variation is implicitly taken into account at each time step and all quantities which influence it (heave, pitch, roll, wave elevation, ...) are included. This procedure implies the remeshing of the hydrodynamic model up to the intersection of the ship with the incident wave elevation at each time instant.

3 NUMERICAL RESULTS

The above described numerical model was used for comparisons with the results of model tests for a 300m container vessel. Three different headings were considered: head waves (180 deg), bow quartering waves \((\beta = 225\) deg) and stern quartering waves \((\beta = 45\) deg). Only regular wave cases are considered. The results for roll motion are presented in Figure 5. The roll damping was estimated by the roll decay tests and corrected according to BV experience. It was subdivided into linear and nonlinear parts and kept the same for all the simulations. The total amount of equivalent linear damping was about 10% of critical damping. It is interesting to note that the final amplitude of roll motion is not very sensitive to the variation of the roll damping value. However, the duration and the characteristics of the transitory phase are strongly affected. Indeed, the nonlinear nature of damping makes the parametric roll to happen earlier.

Anyway, as we can see, the proposed numerical model is able to reproduce the experimental results with very good accuracy so that we conclude the validity of the present approach.

4 CONCLUSIONS

We presented here the numerical model dedicated to the assessment of the parametric roll phenomena. The comparisons of the numerical results with experimental ones shows that this model is very accurate. This might appear quite surprising because the Froude-Krylov approximation, which is the base of the proposed numerical model, is quite crude approximation of the fully nonlinear problem. However, we should keep in mind that the operating conditions which lead to the parametric roll are rather special. Indeed, the wave length is very long (approximately equal to ship length) and ship is sailing close to the head or following waves, which means that the diffraction and radiation phenomena will not be very strong. This imply that the main wave
Figure 5: Roll motion for different headings.
effects will be included in the incident wave field which is rather correctly covered by the Froude-Krylov approximation. It is important to note this fact, because it will be dangerous to expect the same kind of good comparisons for some other nonlinear problems in which the Froude-Krylov forcing is not dominant and other phenomena (diffraction, free surface nonlinearities, ...) are more important.

Finally we should note that the numerical model which is proposed here is very CPU time consuming and its extension to irregular wave field will lead to very expensive numerical simulations. This is mainly due to the remeshing of the model at each time step. For that reason it might be useful to develop more approximate models which can be based on the so called second order approach or eventually on precalculation of the $GM$ for all possible heave-pitch-roll-wave combinations.

REFERENCES


